SENSITIVITY ANALYSIS OF A CASCADE RLS-LMS ALGORITHM FOR DIFFERENT RESOLUTION AUDIO SIGNALS

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ABSTRACT

In this paper, we analyze the effect of both correlated and uncorrelated input signal on the performance of a cascade RLS-LMS based adaptive linear prediction algorithm for lossless compression of different resolution audio signals. We show that the sensitivity of the cascade RLS-LMS predictor to the input signal depends mainly on that of the RLS pre-whitener. Theoretical analysis shows that the mean deviation from the optimum error power is the same for both correlated and uncorrelated input samples and increases with the signal variance, which may lead to a degraded prediction gain of the RLS algorithm for high resolution audio signals. For real audio signals quantized at 16, 20, 24 bit, simulation results confirm our theoretical analysis.

1. INTRODUCTION

Recently, more and more interests focus on lossless coding of high quality audio signal as the broadband services emerge rapidly. Various compression techniques have been proposed for digital audio waveforms. All of the techniques remove firstly redundancy from signal and then code the resulting signal with an efficient digital coding scheme.

The bandwidth constraint applications emphasize the compression ratio over complexity. Considering the characterization of audio signals, abundant tonal and harmonic components and non-stationary, a higher order adaptive FIR predictor possessing good tracking capability is an attractive candidate for audio signal modelling.

There is some research work using adaptive linear prediction for speech prediction [2] and lossless audio coding [3, 4]. A cascade RLS-LMS based adaptive linear prediction algorithm has been reported, which features good coding gain resulting in a better overall compression performance than some widely used lossless audio codecs for audio clips sampling at 44.1 kHz and quantized at 16 bit [4]. However, the challenge for lossless audio coding is that it should provide high compression ratio for storage of audio signals at higher resolution (e.g., 16, 20 and 24 bit) and different sampling rates (e.g., 48, 96 and 192 kHz). One question may be asked: May the RLS-LMS predictor still gives us good prediction gain for audio clips quantized at 20 and 24 bit? We shall show that the sensitivity of the RLS algorithm to the input signal determines the sensitivity of the whole cascade RLS-LMS predictor, since the low-order RLS pre-whitener prior to the LMS algorithm is to reduce the eigenvalue spread of the input covariance matrix to the LMS predictor. The effects of input signals on the performance of RLS algorithm is indirectly through the study of the sensitivity of the RLS algorithm to perturbation in the filter coefficients. In fact, the higher resolution, the more precision the audio signals are represented, e.g., an audio sample quantized at 16 bit, its value is 19060; at 20 bit, its value is 19060.3750, and at 24 bit, its value is 19060.40625. The small change in precision of the samples leads to random perturbation to the filter coefficients. Hence, we focus on the sensitivity of the RLS algorithm for the mean of the deviation from the optimum error power due to random perturbation in the filter coefficient. It was shown that the mean deviation from the optimum error power is the same for both correlated and uncorrelated input samples and increases with the signal variance, which may lead to a degraded prediction gain of the RLS algorithm for high resolution audio signals.

The organization of paper is as follows. In Section 2, we present the RLS-LMS structure and show that the sensitivity of the whole cascade RLS-LMS predictor to inputs depends mainly on the sensitivity of the RLS algorithm to inputs. In Section 3, we study the effects of input correlation on the performance of the RLS algorithm. In Section 4, we show the simulation results to confirm our analysis. Finally, we point out our conclusion and indicate our future work direction.
2. THE CASCADED RLS-LMS ALGORITHM

2.1. Structure
The structure of the cascade RLS-LMS predictor [4] to be studied is shown in Fig. 1. With \( x(n) \) denoting the input to the predictor, the residual error \( x_f(n) \) of the RLS predictor is given by

\[
x_f(n) = x(n) - w^T(n)X(n)
\]

where \( T \) denotes the transpose operator and \( X(n) = [x(n-1), x(n-2), \cdots, x(n-N)]^T \). \( N \) is the number of prewhitener taps. The filter tap weights \( w(n) = [w_1(n), w_2(n), \cdots, w_N(n)]^T \) is updated using the RLS algorithm as follows:

\[
k(n) = \frac{\lambda^{-1}Q(n-1)X(n)}{1 + \lambda^{-1}X^T(n)Q(n-1)X(n)}
\]

\[
w(n) = w(n-1) + k(n)x_f(n)
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\]

and the inverse correlation matrix is

\[
Q(n) = \lambda^{-1}Q(n-1) - \lambda^{-1}k(n)X^T(n)Q(n-1)
\]

where \( \lambda \) is a positive value that is slightly smaller than 1, and initialization of the inverse correlation matrix is

\[
Q(0) = \delta^{-1}I
\]

where \( \delta \ll 1 \). The \( x_f(n) \) is fed to the LMS predictor to get the prediction error \( e(n) \):

\[
e(n) = x_f(n) - p^T(n)X_f(n)
\]

where \( X_f(n) = [x_f(n-1), x_f(n-2), \cdots, x_f(n-M)]^T \), \( M \) is the number of predictor taps. The filter tap weights \( p(n) = [p_1(n), p_2(n), \cdots, p_M(n)]^T \) is updated using the normalized LMS algorithm:

\[
p(n+1) = p(n) + \frac{e(n)}{1 + \mu \| X_f^T(n)X_f(n) \|} X_f(n)
\]

Here we use a special case of the normalized LMS [1], i.e. the parameter \( \mu \) can be turned to trade off adaptation speed and accuracy. Our experience shows that \( 8 < \mu < 120 \) works well for our testing audio clips. Hence, the pre-whitener affects the LMS predictor adaptation algorithm. Thus, the RLS whitener and the LMS predictor depends closely in statistical and algebraical.

2.2. Input Data Model
The input \( x(n) \) is modelled as a random process. We adopt direct-averaging method described in Kushner [5]. The average of the signal \( x(n) \) is equal to the ensemble average. The correlation matrix of the pre-whitener input signal is

\[
R_{xx} = E[x(n)X^T(n)]
\]

Let \( \lambda_{max} \) and \( \lambda_{min} \) are the largest eigenvalue and the smallest eigenvalue of \( R_{xx} \) respectively, the eigenvalue spread is

\[
\chi(R) = \frac{\lambda_{max}}{\lambda_{min}}
\]

The equation (9) describes the ill-conditioning for the input to the LMS predictor. When there is no pre-whitener, if \( \frac{\lambda_{max}}{\lambda_{min}} \gg 1 \), then the input covariance matrix is ill-conditioned, the LMS adaptation is slow. With the pre-whitener, the LMS input is the output \( x_f(n) \) of the RLS pre-whitener with input correlation matrix

\[
R_{xf} = E[x_f(n)X_f^T(n)]
\]

If the RLS pre-whitener has less sensitivity to the perturbation of input signals in the filter coefficients, the eigenvalue spread of the input covariance matrix \( R_{xf} \) to LMS predictor has less sensitivity to the inputs \( x(n) \). Otherwise, the eigenvalue spread of the input covariance matrix \( R_{xf} \) increases, which may degrade the performance of the LMS algorithm. Therefore, the sensitivity of the RLS pre-whitener to the input signals dominates the sensitivity of the whole predictor.

3. STABILITY ANALYSIS OF THE RLS PREDICTOR WITH CORRELATED INPUTS
Since the first order derivatives are zero for the optimal RLS predictor, we can expand the deviation from the optimum error power due to random perturbation in the filter coefficient in a Taylor series with second order partial derivatives to study the sensitivity of the RLS algorithm to inputs. This method has been applied to a system identification problem for \( (\lambda = 1) \) RLS algorithm [6]. Here we use it to adaptive linear prediction problem and analyze \( (\lambda < 1) \) RLS algorithm with uncorrelated/uncorrelated input.

Consider the adaptive linear prediction problem of estimating the desired response \( x(n) \) by forming a linear combination of the current and previous input samples \( x(n) \) to produce the residual error \( x_f(n) \) in Eq. (1). The vector \( x_n \) presents all samples of \( x(n) \) as

\[
x_n = [x(n), x(n-1), \cdots, x(0)]^T
\]

Define the \( n \times N \) matrix, \( X_{N,n} \)

\[
X_{N,n} = [x_n, x_{n-1}, \cdots, x_{n-N+1}]
\]
where
\[ x_{n-1} = z x_n = [x(n-1), x(n-2), \cdots, x(0)]^T \] (13)
and \( z^{-1} \) is the delay unit. The weight matrix \( w_{N,n} \) can be used to estimate \( \hat{x}(n) \) of \( x(n) \) by a linear combination of the current and past input samples. Then the prediction residual error
\[ x_f = x_n^T - X_{N,n} w_{N,n} \] (14)
We define \( \epsilon_N(n) \) - the residual error power of the RLS predictor as a performance measure,
\[ \epsilon_N(n) = X_f(n)X_f^T(n) \] (15)
The optimal RLS filter minimizing the optimum error power is given by [6]
\[ w_{N,n} = x_n^T X_{N,n}(X_{N,n}^T X_{N,n})^{-1} \] (16)
As the first order derivatives of \( \epsilon_N(n) \) with respective to the weight vector are zero for the optimal RLS filter, therefore, consider the performance measure \( J(w_{o}, w_1, \cdots, w_{N-1}) \) where \( \partial J/\partial w_i, i = 0, \cdots, N - 1 \). Then, the second order derivatives w.r.t. due to small perturbations \( \Delta w_i \) is
\[ \Delta J \approx \frac{1}{2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \left( \frac{\partial J}{\partial w_i \partial w_j} \right) |_{\infty} \Delta w_i \Delta w_j \] (17)
Assume that the perturbations \( \Delta w_i \) are zero mean, independent random variables. Hence the mean value \( E\{\Delta J\} \) is
\[ E\{\Delta J\} = \frac{1}{2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \left( \frac{\partial J}{\partial w_i \partial w_j} \right) |_{\infty} E\{\Delta w_i \Delta w_j\} \] (18)
Define \( \varepsilon_i = (\Delta w_i)^2 \) and \( E\{\varepsilon_i\} = \varepsilon \) [6]. Thus
\[ E\{\Delta J\} = \frac{1}{2} \sum_{i=0}^{N-1} \left( \frac{\partial J}{\partial w_i \partial w_i} \right) |_{\infty} \varepsilon \] (19)
Assume \( \epsilon_N(n) \) is the performance measure of the optimal RLS algorithm, which is minimized, we define the deviation from \( \epsilon_N(n) \) as there is the value \( \Delta \epsilon(n) \) due to perturbations:
\[ \epsilon_N(n) = \epsilon_N(n) + \Delta \epsilon(n) \] (20)
Now we try to derive the mean value of this deviation from the optimum error power since the perturbations are random variables, \( \Delta \epsilon(n) \) is random. From (14) and (15), we can get
\[ \frac{\partial^2 \epsilon_N(n)}{\partial w_{N,n}^2} = 2 X_{N,n}^T X_{N,n} \] (21)
Define the matrix \( \Phi(n) \)
\[ \Phi(n) = X_{N,n}^T X_{N,n} \] (22)
which describes the sensitivity of the algorithm to perturbation in the weight coefficients. For our adopted RLS algorithm
\[ \Phi(n) = X_{N,n}^T X_{N,n} = \sum_{i=0}^{n} \lambda^{n-i} x(i)x^T(i) \] (23)
For \( n \) becomes large, we have
\[ E\{x(i)x^T(i)\} = R_x \] (24)
where \( R_x \) is the sample covariance matrix. Since \( \lambda < 1 \), hence
\[ E\{\Phi(n)\} = \sum_{i=0}^{n} \lambda^{n-i} R_x = \frac{1 - \lambda^{n+1}}{1 - \lambda} R_x \] (25)
If \( x(n) \) is a white random process
\[ R_x = \sigma_x^2 I \] (26)
The mean value of the error power deviation due to random perturbation \( \Delta \epsilon(n) \) in the weights \( w_{i}(n) \) was derived as [6] for \( n \) large, since \( \lambda < 1 \):
\[ E\{\Delta \epsilon(n)\} = \frac{1}{1 - \lambda} N \varepsilon \sigma_x^2 \] (27)
where \( \varepsilon = E[\Delta \epsilon(n)] \) and \( \sigma_x^2 \) is the variance of the input samples, \( N \) is the filter order.
For correlated signals \( R_x \), we have
\[ R_x = \begin{bmatrix} r_x(0) & r_x(1) & \cdots & r_x(N - 1) \\ r_x(1) & r_x(0) & \cdots & r_x(N - 2) \\ \vdots & \vdots & \ddots & \vdots \\ r_x(N - 1) & \cdots & \cdots & r_x(0) \end{bmatrix} \] (28)
of which each element \( r_x(i) \) is
\[ r_x(i) = E\{x(n)x(n - i)\} \] (29)
In particular
\[ r_x(0) = \sigma_x^2 \] (30)
Since \( R_x \) is symmetric, from (18), (25), we have the mean value of the deviation from the optimum error power (20):
\[ E\{\Delta \epsilon(n)\} = \frac{1}{1 - \lambda} N \varepsilon \sigma_x^2 \] (31)
The mean of the deviation from the optimum error power is the same for equal power of uncorrelated and correlated signals Eq.(27) and Eq.(31). From the formula, we can see that the mean of deviation from the optimum error power increases with the variance of the input \( \sigma_x^2 \). It may lead to high eigenvalue spread of input metrix to the LMS predictor, resulting slow convergence. For the audio clips quantized at 16, 20, 24 bit, the higher resolution may be the larger variance of inputs. The performance of the cascade RLS-LMS may be degraded with high resolution inputs.
This paper analyzes the effect of the inputs on the performance of the RLS-LMS predictor for lossless audio compression. The analysis shows that the mean of the deviation from the misadjustment (steady-state MSE) is the same for equal power correlated and white signals, but increases with the variance of signal, which leads to the degraded performance of the RLS-LMS predictor for high resolution signals. Experimental results of real audio data verified our theoretical analysis. In order to provide a work rule for practical turning parameters of the RLS-LMS predictor, we are studying the transient behavior, the steady-state MSE, the performance bound of the RLS-LMS predictor, which may demonstrate that the RLS-LMS may outperform the corresponding Wiener filter.

4. SIMULATION RESULTS

We carried out some experimental work to evaluate the performance of the RLS-LMS predictor for real audio data. In the first experimental work, we evaluate the prediction gain for audio signals sampled at 48 kHz and quantized at 16, 20, 24 bit respectively. The results are given in Table 1. The coding gain decreases as the resolution of audio signal increases, which confirms our theoretical analysis.

In the second experimental work, we compare the RLS-LMS predictor with predictors of the benchmark lossless codecs: Monkey 3.97 [8] and MPEG-4 CE2 provided by the Technical University of Berlin (TUB) [7] for two training sets: 48 kHz/16 bit and 192 kHz/24 bit. The results are given in Table 2. The RLS-LMS predictor gives the best prediction gain for the signal (48 kHz/16 bit).

Finally, we integrate the RLS-LMS predictor into MPEG-4 CE2 lossless audio codec to evaluate its practical performance. In fact, we have tested 108 audio clips. Now we present the results for 2 audio files, which are stereo, sampled at 48 kHz and quantized at 16, 20, 24 bit. We compare the compression ratio of the RLS-LMS codec with the codecs of MPEG-4 CE2 (TUB) and Monkey’s audio compression under their highest compression ratio setting. The compression ratio is defined as

\[ C = \frac{\text{Original file size}}{\text{Compressed file size}} \times 100 \]  \hspace{1cm} (32)

The results are given in Table 3. The table 3 shows that the RLS-LMS lossless codec degrades the compression ratio with high resolution audio. Clearly, the best compression performance is achieved by the RLS-LMS codec, with average improvement 0.91% and 2.53% compared to Monkey’s and TUB codecs, respectively.

5. CONCLUSION

This paper analyzes the effect of the inputs on the performance of the RLS-LMS predictor for lossless audio compression. The analysis shows that the mean of the deviation from the misadjustment (steady-state MSE) is the same for equal power correlated and white signals, but increases with the variance of signal, which leads to the degraded performance of the RLS-LMS predictor for high resolution signals. Experimental results of real audio data verified our theoretical analysis. In order to provide a work rule for practical turning parameters of the RLS-LMS predictor, we are studying the transient behavior, the steady-state MSE, the performance bound of the RLS-LMS predictor, which may demonstrate that the RLS-LMS may outperform the corresponding Wiener filter.

6. REFERENCES