IMAGE RETRIEVAL BASED ON 2-D HISTOGRAM OF FRACTAL PARAMETERS

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ABSTRACT

An image can be characterized by its fractal parameters, and hence, the fractal parameters can be used as the image signature to retrieve the images. In this paper, based on the principle that fractal transform is completely determined by luminance offset and contrast scaling, we first propose histogram of luminance offset as a statistical index, and we further propose three composite indices by combining individual histograms to enhance retrieval rate and reduce computational complexity. Experimental results on a database of 416 texture images indicate that the proposed indices significantly improve the retrieval rate, compared to other retrieval methods.

1. INTRODUCTION

Fractal image coding [1-3] originally developed by Barnsley et al. [1], is based on approximating an image by an attractor of an Iterated Function System (IFS). Subsequently, Jacquin implemented a block-based fractal compression scheme by Partition IFS (PIFS), which is popularly known as fractal block coding [2]. The encoding of each range block consists of finding the “best-pair” domain block in the domain block pool.

An image can be characterized by its fractal parameters, hence, the fractal parameters can be used as the image signature to retrieve an image from image databases. As a result, a few fractal indexing techniques have been proposed recently.

Zhang et al. [5] have proposed a fractal indexing technique where the fractal codes are used as the image index (referred to as the FC technique). However, because the complexity of direct difference of the fractal codes is proportional to the size of image, the FC technique cannot provide fast search. In addition, because of the large collage error, such fractal codes cannot be used to reconstruct the retrieved image. Hence the indexing technique is not used in the fractal compression domain.

Julie et al. [6] have proposed two major attributes as the image index: mean of contrast scaling parameters and mean of the luminance offsets. Although, this technique provides a good indexing performance, the complexity is very high.

Schouten et al. [7] proposed to employ histogram of contrast scaling parameters as an image index (referred to as the HWQCS technique). Although the retrieval is very fast, the index technique does not provide a high retrieval rate.

Because fractal transform is completely determined by luminance offset and contrast scaling, any indexing technique based on the individual histograms is incomplete and cannot provide high retrieval rate. Based on this principle, we first propose histogram of luminance offset as a statistical index, and we further propose three composite indices by combining individual histograms. Experimental results indicate that the proposed indices provide good retrieval performance.

The remainder of the paper is organized as follows. Section 2 reviews fractal coding and the HWQCS technique. The proposed indices are presented in Section 3. Experimental results are reported in Section 4, which is followed by the conclusions.

2. FRACTAL CODING AND THE HWQCS TECHNIQUE

In the section, we first present a brief review of fractal block coding [2-3], and then introduce the HWQCS technique [7].

2.1 Fractal Coding

For each range block $R = \{r_i\}$, traditional fractal block coding seeks to minimize the following distortion:

$$E(R, D) = \|R - \tau(D)\| = \|R - sD - gU\| + \sum (sd_i + g - r_i)$$

(1)

over pre-contractive $D = \{d_i\} \in \Omega$ ( $\Omega$ is the pre-contractive domain block pool) with respect to the contrast scaling parameter $s$ and luminance offset $g$. Note that in Eq. (1), $U$ is a matrix whose elements are all ones and $\| \|$
is the 2-norm. Given a pre-contractive domain block $D$, the optimal $s$ and $g$ (in the least squares sense) can be obtained as follows [3]:

$$s = < R - \tau U, D - \bar{D} > / \| D - \bar{D} \| , \quad g = \tau - s \bar{D}$$ (2)

where $\tau$ and $\bar{D}$ are the average intensities of the range blocks and the pre-contractive domain blocks, respectively.

Because quantization of the optimal parameters increases collage error, and causes the decoding image blurred, it is better to work with a set of the pre-quantized fractal parameters $\{s_j\}_{j=1}^J$ and $\{g_i\}_{i=1}^I$ [4]. Note that the indices $I$ and $J$ correspond to the quantized number for $g$ and $s$ respectively. Thus, the optimal $s$ and $g$ are obtained by minimizing the following distortion:

$$\hat{E}(R, D) = \| R - g_j U - s_i D \|^2 = \| R - \tau U \| + s_i \| D - \bar{D} \|^2 + 2(\tau - s_i \bar{D} - g_j)^2$$

$$- 2s_i \langle R - \tau U, D - \bar{D} \rangle$$ (3)

over $D \in \Omega$. The fractal code of $R$ is

$$(g, s, x_D, y_D) = \arg \min_{D \in \Omega} \hat{E}(R, D)$$

where $(x_D, y_D)$ is top-left corner coordinate of the “best pair” domain block.

### 2.2 The HWQCS technique

Schouten et al. [7] proposed HWQCS technique where the histogram of weighted quad-tree contrast scaling parameter $s$ is used as the image index. If $L$ refers to the depth of quad-tree partition and $J$ refers to the quantized level for $s$, $\{v_j\}_{j=1}^J$ refers to the normalized histogram of $s$ corresponding to level $l$ ($1 \leq l \leq L$), then the image index used in [7] is

$$h_{lj} = w_j v_j \quad (1 \leq l \leq L, j = 1, \ldots, J)$$

where $w_j$ is the weighting factor corresponding to level $l$. For the single fractal coding, the index $h_{lj}$ degrades into the histogram of contrast scalings. Although, the HWQCS technique is fast, the retrieval rate is not high enough.

### 3. THE PROPOSED INDEXING TECHNIQUE

Since the HWQCS technique only exploits the individual histogram, the retrieval rate is not high enough. In fact, fractal transform is completely determined by luminance offset and contrast scaling, any indexing technique based on the individual histograms is incomplete and cannot provide high retrieval rate. Hence, we propose three composite indices by combining individual histograms. These indices are described in detail below.

#### 3.1 Indices

**Index-1: Histogram of luminance offsets (HLO)**

The HWQCS technique highlights the significance of contrast scaling, but ignores the luminance offset. In fact, the luminance offset is also an important fractal parameter. The normalized histogram of luminance offsets $\{g_i\}_{i=1}^I$ is denoted as $\{p(g_i)\}_{i=1}^I$. However, the retrieval rate is not high enough only using histogram of luminance offsets as the index.

**Index-2: 2D joint histogram (2DH)**

Since the fractal transform $\tau$ is determined by both $g_i$ and $s_j$, in theory, the 2-D joint normalized histogram of $g_i$ and $s_j$ efficiently captures statistical features of texture images, which is expressed as:

$$\{q(g_i, s_j)\} \quad (i = 1, \ldots, I; \; j = 1, \ldots, J)$$

The 2-D joint histogram provides finer feature representation than individual histograms of $g_i$ or $s_j$. Hence, it is expected to significantly improve retrieval rate.

**Index-3: Tensor product of individual histograms (TPH)**

An approximation to the 2D joint histogram is tensor product of histograms of luminance offsets and of contrast scalings, which is expressed as:

$$\{p(g_i)\}_{i=1}^I \otimes \{v_j\}_{j=1}^J = \{p(g_i)\}_{i=1}^I \{v_j\}_{j=1}^J$$

Tensor product decreases the storage space, but does not reduce computational complexity.

**Index-4: Weighted average of individual histograms (WAH)**

In theory, the 2D joint histogram is an optimal statistical index. However, its dimension is greatly expanded. Tensor product only reduces storage space, but does not reduce computational complexity. To avoid rapid growth in dimension, we construct WAH by weighting individual histograms, which is expressed as:

$$w \{p(g_i)\}_{i=0}^I + (1 - w) \{v_j\}_{j=1}^J$$

where $w$ and $(1-w)$ are the weights of histograms of luminance offsets and histogram of contrast scalings, respectively.
3.2 Weight estimation

WAH combines individual histograms by a weighted average, and hence the retrieval rate is superior to that of using individual histograms. Our concern is focused on selecting the optimal weight to maximize the retrieval rate close to that of the 2D histogram.

Obviously, the weights depend on the retrieval rates of individual histograms. A high retrieval rate means high credit and large weight. Based on this principle, we propose a simple weight estimation approach. If $c_s$ and $c_g$ are the retrieval rates of histograms of $s$ and $g$, respectively, then the weight of histogram of $g$ is estimated as follows:

$$w = \frac{c_g}{c_s + c_g}$$

Our experimental results testify that the weight estimated by (4) is very close to the optimal weight, and the retrieval rate approaches that of a 2D joint histogram.

3.3 Similarity Measurement

To measure the similarity between the query image and the candidate images, the proposed indices must be matched using a distance criterion. In this paper, we choose Manhattan distance as the metric. If $H_q = \{h_0, h_1, \ldots, h_{V-1}\}$ and $H_c = \{\hat{h}_0, \hat{h}_1, \ldots, \hat{h}_{V-1}\}$ are the histograms of the query image and candidate image, respectively, the Manhattan distance between the two images is calculated as follows:

$$d(Q, C) = \sum_{i=0}^{V-1} |h_i - \hat{h}_i|$$

where $V$ is the number of bins.

Fig. 1 shows four similar and four dissimilar texture images compared to (a). The Manhattan distances corresponding to these images are shown in Fig 2. In general, the distances are small for similar texture images, and large for the different texture images.

4. PERFORMANCE EVALUATION

In this section, we present the performance of the proposed indices and compare it with other retrieval methods.

We have used twenty-six 512x512 gray-scale Brodatz texture images. Each of the 512x512 image is divided into sixteen 128x128 non-overlapping subimages to create a test database of $Z=416$ texture images. Each subimage is fractal encoded using exhaustive search and $g$ is quantized to 6 bits, and $s$ is quantized to 2 bits or 3 bits. Ideally, all sixteen images, corresponding to a selected test image, should be retrieved in each test. However, this does not generally happen in practice. We evaluate the performance of the proposed and other retrieval methods using the average retrieval rate [7] that is defined as follows. Let $F$ be the number of ideally retrieved images (in this case $F=16$), and $m_z$ be the number of correctly retrieved images at $z$-th test. The average retrieval rate is then calculated as:

$$\text{Average retrieval rate} = \frac{1}{Z} \sum_{z=1}^{Z} m_z / (F \times Z)$$

In order to compare the proposed techniques with existing fractal indexing techniques, we implemented two typical techniques: FC [5] and HWQCS [7].

The average retrieval rates of various techniques are shown in Table 1. The average retrieval rate of FC is 21.4%. Note that the FC technique does not exploit statistical similarity of the texture images, and hence the retrieval rates are very low.

The average retrieval rate of HWQCS is 45.9 (2 bits) and 47.9 (3 bits). It is observed that the average retrieval rate is enhanced only marginally after increasing quantization level for $s$.
The retrieval rate of HLO is close to that of the HWQCS technique. It demonstrates that the luminance offset is also an important fractal feature.

As we expect, 2DH provides the highest retrieval rate (71.6%) amongst the proposed indices, and greatly improves retrieval rates of individual histograms. TPH is a good approximation to 2DH, and its retrieval rate (69%) is close to the retrieval rate of 2DH.

The retrieval rates of WAH for different weights are shown in Fig. 3. The retrieval rate peaks around \( w = 0.5 \), which is very close to the weight estimated by Eq. (4) \( (w = 0.52) \).

**Fig. 3:** Retrieval rate vs. weights

![Graph showing retrieval rate vs. weights](image)

**Table 1.** Average Retrieval Rate (ARR) of different retrieval methods. MxN is the size of image, and BxB is the size of a range block.

<table>
<thead>
<tr>
<th>Retrieval Method</th>
<th>Length of the feature vector</th>
<th>ARR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC [5]</td>
<td>( 4 \times (M/B) \times (N/B) )</td>
<td>21.4</td>
</tr>
<tr>
<td>GGD-KLD [8]</td>
<td>18</td>
<td>69.9</td>
</tr>
<tr>
<td>HWQCS [7] ((L = 1))</td>
<td>4 (2 bits)</td>
<td>45.9</td>
</tr>
<tr>
<td></td>
<td>8 (3 bits)</td>
<td>47.9</td>
</tr>
<tr>
<td>HLO</td>
<td>64</td>
<td>49.4</td>
</tr>
<tr>
<td>2DH</td>
<td>256</td>
<td>71.6</td>
</tr>
<tr>
<td>TPH</td>
<td>256</td>
<td>69.0</td>
</tr>
<tr>
<td>WAH ((w = 0.52))</td>
<td>68</td>
<td>68.0</td>
</tr>
</tbody>
</table>

WAH has a performance comparable to GGD-KLD technique that employs wavelet-based features [8]. Computational complexity is reduced by a two-step retrieval method:

**Step 1:** Select a short list of candidate images by matching histogram of contrast scaling.

**Step 2:** Retrieval the top \( \mu \) “closest” images from the short-list images by matching WAH.

Fig 4 shows a query image and the top sixteen retrieved images using the HWQCS technique and 2DH, respectively. Fig 4(a) is the query image. Fig 4(b) is the top 16 retrieved images using the HWQCS technique (2 bits); note that six visually different texture images are retrieved. However, using 2DH, only two visually different images are left in the top 16 images (Fig 4(c)).

**Fig.4:** The query image and the top retrieved images

![Image showing query and retrieved images](image)

**5. CONCLUSIONS**

In this paper, based on the principle that fractal transform is determined by contrast scaling and luminance offset, we proposed three composite indices to enhance retrieval rate and reduce computational complexity. Experimental results testify our expectation and show that the proposed technique provides superior performance. Although our discussion is focused on the single level fractal block coding, we can extend the proposed indexing technique into multilevel fractal block coding by considering a multilevel histogram.

![Query image](image)

![Retrieved images using HWQCS](image)

![Retrieved images using 2DH](image)

**REFERENCES**


