Performance Analyses of Coherent Fast Frequency-Hopping Spread-Spectrum Systems with Partial Band Noise Jamming and AWGN

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Abstract
In this paper, the performance analysis of a coherent fast frequency-hopping spread-spectrum (FHSS) system employing binary phase-shift-keying (BPSK) with the presence of partial band noise jamming (PBNJ) and additive white Gaussian noise (AWGN) is extended to multiple hops per signalling interval. The bit error rate (BER) expressions for the coherent maximum likelihood (ML), linear combination (LC) and hard decision majority vote (HDMV) receivers in such systems are derived and validated by the simulation results. It is shown that under the worst-case PBNJ condition the coherent ML receiver offers the optimal performance. The coherent LC receiver is incapable of providing diversity improvement under the worst-case PBNJ condition. In contrast, the coherent HDMV receiver provides significant diversity improvement at moderate signal-to-jamming ratio (SJR).

1. Introduction
The performance of coherent fast frequency-hopping spread-spectrum (FHSS) systems operating in the presence of partial band noise jamming (PBNJ) and additive white Gaussian noise (AWGN) with one hop per signalling interval has already been examined in [1]-[4]. In [1], based on the assumption of perfect time and phase synchronization, the performance of a coherent FHSS system with quadrature modulations under the worst-case PBNJ was presented. [2]-[4] analysed the performance degradation due to imperfect synchronization. In order to fully recover the non-coherent demodulation and combination loss, a frequency diversity scheme was recently proposed for a phase-coherent FHSS system with PBNJ [5].

In this paper, we extend the performance analysis of a coherent fast FHSS system employing binary-phase-shift-keying (BPSK) in the presence of PBNJ and AWGN to multiple hops per signalling interval with three types of coherent receivers, namely, maximum likelihood (ML) receiver, linear combination (LC) receiver, and hard decision majority vote (HDMV) receiver. Similar to [5], we restrict the bandwidth inside which the communicator signal can hop in order to maintain phase continuity. Perfect time and phase synchronization are assumed throughout this paper.

2. System Model
At the transmitter of the coherent fast FHSS system employing BPSK modulation, the incoming data is first modulated by mapping the binary data sequence of 0s and 1s to a sequence of antipodal waveforms \( s_i(t) \) and \( s_0(t) \), respectively, with a rate of \( R_0 = 1/T_b \), where \( T_b \) is the bit duration. The modulated signal is then hopped to one of \( N \) non-overlapping frequency-hopping bands in a pseudorandom manner with a rate of \( R_h = 1/\tau \), where \( \tau = \tau_0 / L \) is the hopping duration, and \( L \) is the diversity level. The resultant signal is band-pass filtered and up-converted for transmission. At the receiver, the desired signal corrupted by AWGN and PBNJ is down-converted and band-pass filtered. It is then dehopped by a dehopper to convert the desired signal back to the base-band.

The received base-band signal for the \( l \)th hop interval of a particular data bit after dehopping process can be expressed as

\[
\begin{align*}
\tilde{r}_l(t) &= s_i(t) + w_i(t) + j_i(t) \quad (l - 1)T_b \leq t \leq lT_b \\
\end{align*}
\]

where \( s_i(t) \) is the desired signal, \( w_i(t) \) is the equivalent base-band noise, and \( j_i(t) \) is the equivalent base-band jamming signal. The desired signal \( s_i(t) \) is given by

\[
\begin{align*}
s_i(t) &= \sqrt{E_i / T_i} \cos(2\pi f_0 t + \pi) \\
\end{align*}
\]

where \( E_i \) is the bit energy, \( f_0 \) is the base-band frequency of desired signal. The background noise \( w_i(t) \) is assumed to be Gaussian distributed with a single-sided power spectral density (PSD) of \( N_0 \) Watts/Hz. The jamming signal \( j_i(t) \) is modelled as a Gaussian random process and it is assumed to be active on a fraction \( \eta \) of the \( N \)
frequency-hopping bands. As a result, when the jammer is present, the effective single-sided PSD of the jammer is given by $N_j/\eta$ Watts/Hz, where $N_j$ is the equivalent single-sided broadband jamming PSD defined as the ratio of total jamming power $P_{jt}$ to the total system bandwidth $W_s$. By defining a random variable $q_l = 1$ with probability of $\eta$ and $q_l = 0$ with probability of $(1-\eta)$ to represent the jamming state on the $l$th hop, we can express the single-sided PSD of the noise and jammer for the $l$th hop interval of an observation data bit in a compact form as follows:

$$N_l = N_s + q_l N_j/\eta, \text{ } l = 1, 2, ..., L$$

(3)

3. Performance Analyses

3.1 Coherent Maximum Likelihood (ML) Receiver

The equivalent base-band signal $r(t)$ can be viewed as the desired signal $s(t)$ corrupted by a zero-mean Gaussian random process with a single-sided PSD of $N_j$. The probability density function (PDF) of $r(t)$, assuming $s(t)$ was transmitted, can therefore be expressed as [6]

$$p_r(r | s) = C_j \exp \left( -\frac{1}{N_j} \int [r(t) - s(t)]^2 dt \right)$$

(4)

where $C_j$ is a normalized constant [6]. Assuming the diversity receptions from different hops are statistically independent of each other, the joint PDF of the received signal from $L$ hops is the product of the PDFs of all the hops, which can be shown to be

$$p_r(r | s) = \prod_{h=1}^{L} C \exp \left( -\frac{1}{N_j} \int [r_h(t) - s(t)]^2 dt \right)$$

(5)

In the coherent fast FHSS system employing BPSK modulation, with the likelihood of input messages being equal and the energy per hop being the same regardless of the message transmitted, the ML detector will decide on $s(t)$ that makes the joint PDF of (5) larger. Equivalently, by ignoring the message independent terms when comparing $p_r(r | s_1)$ and $p_r(r | s_2)$, we can define the test statistics as follows:

$$\Lambda_{ML} = \sum_{l=0}^{L} \frac{1}{N_j} \int [r_l(t) - s_l(t)]^2 dt$$

(6)

The ML receiver will decide that $s_l(t)$ was transmitted if the test statistics is greater than zero, and vice versa.

Without loss of generality, we assume that $s_l(t)$ was transmitted. Denote $E$ as the energy per hop, the mean and variance of the Gaussian distributed test statistics $\Lambda_{ML}$ given that $s_l(t)$ was transmitted can be shown to be

$$\mu_{ML} = 2E \frac{1}{N_j}, \text{ } \sigma_{ML}^2 = 2E \frac{1}{N_j}$$

(7)

With $E = E_0/L$, the conditional probability of bit error can therefore be expressed in terms of Gaussian Q-function as follows:

$$P_{ml}(e | s_l) = Q \left( \frac{2E_0}{L \eta N_j} \right)$$

(8)

Substitute (3) into (8), we obtained the conditional probability of bit error given that $k$ out $L$ hops have been jammed, expressed as

$$P_{ml}(e | k) = \frac{\sqrt{E_0}}{\sqrt{L \eta N_j}} \int_0^L \frac{L - k + 1}{L - k - 1} \left( 1 - e^{-2E_0/(L \eta N_j)} \right)$$

(9)

The unconditional probability of bit error of the coherent ML receiver can then be obtained by averaging equation (9) over the probability of $k$ out $L$ hops having been jammed. That is,

$$P_{ML}(e) = \sum_{k=0}^{L} \frac{L}{k} \left( 1 - \frac{1}{2L} \right) P_{ml}(e | k)$$

(10)

3.2 Coherent Linear Combination (LC) Receiver

In the optimal receiver structure given in (6) the PSD $N_j$ serves as the weighting factor to the individual hops. If the information on the PSDs is not available, it is possible to simply set all the weighting factors to one, resulting in a coherent LC receiver with the test statistics given by

$$\Lambda_{LC} = \sum_{l=1}^{L} \frac{1}{N_j} \int [r_l(t) - s_l(t)]^2 dt$$

(11)

Similarly, the conditional probability of bit error for the coherent LC receiver can be shown to be

$$P_{lc}(e | s_l) = Q \left( \frac{2E_0}{\sqrt{L \eta N_s}} \right)$$

(12)

Substitute (3) into (12), we obtained the conditional probability of bit error given that $k$ out $L$ hops have been jammed given by

$$P_{lc}(e | k) = \frac{2E_0}{\sqrt{L \eta N_s}} \left( \frac{L - k + 1}{L - k - 1} \right)$$

(13)

The unconditional probability of bit error of the coherent LC receiver can then be obtained by averaging equation (13) over the probability of $k$ out $L$ hops having been jammed. That is,

$$P_{LC}(e) = \sum_{k=0}^{L} \frac{L}{k} \left( 1 - \frac{1}{2L} \right) P_{LC}(e | k)$$

(14)

3.3 Coherent Hard Decision Majority Vote (HDMV) Receiver

A coherent HDMV receiver makes hard decisions for individual hops and then votes on the majority to produce a final decision. The individual hops are coherently detected with probability of bit error given by

$$P_e = \eta Q \left( \frac{2E_0}{\sqrt{L \eta N_s}} \right) + (1-\eta)Q \left( \frac{2E_0}{\sqrt{L \eta N_j}} \right)$$

(15)

The unconditional probability of bit error is then obtained as

$$P_{HDMV}(e) = \sum_{l=0}^{L} \frac{L}{k} P_e^k (1-P_e)^{L-k}, \text{ } L \text{ is odd}$$

$$P_{HDMV}(e) = \sum_{l=0}^{L} \frac{L}{k} P_e^k (1-P_e)^{L-k} + \frac{1}{2L} \int L \left( P_e^k (1-P_e)^{L-k} - P_e^k (1-P_e)^{L-k} \right), \text{ } L \text{ is even}$$

(16)
4. Numerical Results and Discussions

In this section, the analytical and simulated bit error rate (BER) results are presented. The analytical results for the coherent ML, LC and HDMV receivers are obtained from (10), (14) and (16), respectively. The signal-to-noise ratio (SNR) defined as $E_s/N_0$ is fixed at 13.35dB, and the signal-to-jamming ratio (SJR) defined as $E_s/N_j$ is in the range of 0–50dB. The total number of available frequency-hopping band is assumed to be $N=1000$.

Figures 1, 2 and 3 show the BER results with different diversity levels for the coherent ML, LC, and HDMV receiver, respectively. Both the worst-case BER and the BER under the broadband jamming condition are presented and confirmed by the simulation results shown with plus sign. The close match between the analytical results and the simulation results validates the BER expressions presented in the previous section.

From Figure 1, it is observed that the worst-case BER for the coherent ML receiver improves as the diversity level increases. It is clear that there is a huge performance improvement (about 2 decades at SJR=20dB) when the diversity level increases from 1 to 2. When the diversity level goes higher, the additional improvement becomes less. Eventually, there is no more additional performance improvement and the worst-case BER will coincide with the BER under the broadband jamming condition. It should be noted that the BER results for different diversity levels under the broadband jamming condition are the same.

From Figure 2, it is observed that there is no diversity improvement for the coherent LC receiver under the worst-case PBNJ condition. This is expected since the smart jammer will distribute its limited power in less frequency-hopping bands but with a higher effective jamming PSD. As a result, the BER with one hop being jammed, which dominates the average BER for the coherent LC receiver, remains roughly the same. It should be noted that at higher SJR the worst-case BER improves slightly with diversity. This is mainly due to the assumption that the minimum value of the PBNJ fraction is $I/N=0.001$ in this case.

From Figure 3, it is observed that the worst-case performance of the coherent HDMV receiver improves as the odd diversity level increases from 1 to 5 for SJR in the range of 10–35dB. And there is a significant improvement (about 1.5 decades at SJR=20dB) when the diversity level increases from 1 to 3. As the diversity increases further, the additional improvement becomes less. It is also interesting to notice that as the diversity level increases the worst-case BER curve and the curve under the broadband jamming condition are approaching to each other with SJR in the region of 10–35dB.

Figure 4 plots the worst-case BER results of the coherent HDMV receiver against the odd number of diversity levels for different SJRs. From this figure, we can simply find the minimum for a specific curve associated with a particular SJR and then project vertically to obtain the optimal diversity level and project horizontally to obtain the corresponding BER. When all the minimum points were traced, we obtained the locus of the optimal diversity level for a coherent HDMV receiver, which is shown in dotted line of Figure 4. The traced locus helps us to choose an appropriate diversity level for the coherent HDMV receiver.

Figure 5 compares the worst-case performance of the coherent ML, LC and HDMV receivers with that of the non-coherent square-law nonlinear combining receiver with adaptive gain control (AGC) [7], LC receiver [8], and HDMV receiver [9] with $L=3$. It can be observed that the coherent ML, LC and HDMV receivers provide great performance improvement as compared to the non-coherent AGC. LC and HDMV receivers, respectively. It is further observed that the coherent HDMV receiver even gives more than 1 decade of improvement as compared to the non-coherent AGC receiver for SJR in the range of 10–35dB.

5. Conclusions

In this paper, we have extended the performance analysis of a coherent fast FHSS/BPSK system in the presence of PBNJ and AWGN from one hop per signalling interval to multiple hops per signalling interval. We have also derived the BER expressions for such systems incorporated with the coherent ML, LC and HDMV receivers and presented in (10), (14) and (16), respectively.

Based on these BER expressions, we have shown that the coherent ML receiver offers the optimal performance. And there is about 2 decades of performance improvement at SJR=20dB when the diversity level increases from 1 to 2. However, the coherent ML receiver requires the information on the jamming PSD of individual hops and the PSD of AWGN (side information), which can not be obtained exactly in practice. The coherent LC and HDMV receivers do not require the side information and they are relatively easier to be implemented. It is shown that the coherent LC receiver under the worst-case PBNJ condition is incapable of providing diversity improvement. In contrast, we have also shown that the coherent HDMV receiver under the worst-case PBNJ condition provides significant diversity improvement when SJR is in the region of 10–35dB. With SJR in the same region, the HDMV receiver provides greater amount of improvement than the coherent LC receiver against the same type of non-coherent receiver. Therefore, the coherent HDMV receiver is preferred in this case to suppress the PBNJ in a coherent fast FHSS system.

References


