Distortion and Designing of Signals Propagating in a Coaxial Cable

S. K. Foong; C. H. Lim
Natural Sciences Academic Group
National Institute of Education
Nanyang Technological University

Abstract

We study the waveform distortions of a voltage pulse along the transmission line, particularly the decrease in amplitude and dispersion. Based on the solution of the telegrapher’s equation, we propose a simple designing algorithm and generated the designed pulse so that after the distortion, it would appear as a square pulse of the desired amplitude and duration at a designated point on the line. From the experimental data taken at points 50 m, 100 m and 150 m away from the generator, we concluded that a reduction of about 2/3 in the distortions was achieved, with a possibility of further improvement.

Keywords: transmission line, telegrapher’s equation, waveform distortions, waveform designing.

1. Theory and Method

A general solution for the telegrapher’s equation of voltage along a transmission line with line parameters $R, L, C$ and $G$, namely

$$\frac{\partial^2 v(z, t)}{\partial z^2} - LC \frac{\partial^2 v(z, t)}{\partial t^2} + \left( LG + RC \right) \frac{\partial v(z, t)}{\partial t} - R G v(z, t) = 0,$$

and with one end connected to an ac voltage source of frequency $\omega$, is given by

$$v(z, t) = \sum_{n=1}^{\infty} \left[ b_n e^{-\alpha_n z} \cos(\omega t - \beta_n z + \varphi_n^+) + c_n e^{\alpha_n z} \cos(\omega t + \beta_n z + \varphi_n^-) \right], \quad (2)$$

where

$$\alpha = \sqrt{\frac{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)}{2} + RG - \omega^2 LC}$$

$$\beta = \sqrt{\frac{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2) - RG + \omega^2 LC}{2}}.$$

Physically, this corresponds to two sinusoidal waves travelling in opposite directions, each with the same speed $\omega/\beta$ but with an amplitude that decreases exponentially with distance in the direction of propagation. The attenuation factor $\alpha$ and the phase factor $\beta$ are dependent on the frequency $\omega$ and the line parameters, which in general are frequency-dependent [1]. The amplitudes $|b_n^+|, |c_n^-|$ and the phases $\varphi_n^+, \varphi_n^-$ are dependent on the boundary conditions such as the output of the generator situated at one end and the load at the other end of the line.

Suppose a square pulse of duration $l'$ and period $T$, represented by the Fourier series of cosine and sine terms,

$$V(t) = V_0 \sum_{n=1}^{\infty} \left[ b_n \cos(\omega_n t) + c_n \sin(\omega_n t) \right] \quad (3)$$

is generated at the sending end (denoted as $z = 0$) at $t = 0$, and propagated in the +z-direction, the distorted pulse at a point $z$ is given by

$$v(z, t) = V_0 \sum_{n=1}^{\infty} \left[ b_n e^{-\alpha_n z} \cos(\omega_n t - \beta_n z) + c_n e^{\alpha_n z} \sin(\omega_n t - \beta_n z) \right] \quad (4)$$

with $\alpha_n$ and $\beta_n$ refer to $\alpha$ and $\beta$ evaluated at the frequency $\omega_n$. Each of the component wave of frequency $\omega_n$ suffers a different amount of amplitude attenuation. In addition, the pulse changes its form as each wave component travels at a different speed $u_n = \omega_n/\beta_n$. The distorted pulse of Eq.(4) is computed using the frequency-dependent formulae for $R, L$ and $G$ in [1]. Distortion of several digital signals such as rectangular pulse and unmodulated sinusoidal pulse were studied theoretically in [2], without considering the frequency dependence of the line parameters.

In designing the square pulse, consider the pulse $v_T$ which is $v$ of Eq. (4) subjected to a space and time translation as follows:

$$v_T(z, t) = v(z - z_0, t - t_0).$$
\[ v_T(z, t; z_1) = v(z - z_1, t - t_1) \]

\[ = V_0 \sum_{n=1}^{\infty} b_n e^{-\alpha_n(z-z_1)} \times \cos [\omega_n(t - t_1) - \beta_n(z - z_1)] + \]

\[ V_0 \sum_{n=1}^{\infty} c_n e^{-\alpha_n(z-z_1)} \times \sin [\omega_n(t - t_1) - \beta_n(z - z_1)], \]

(5)

where \( z_1 \) is the designated point that we want the square pulse to appear at time \( t_1 = z_1/u_1 \). At the point \( z = z_1 \), we have

\[ v_T(z_1, t; z_1) = v(0, t - t_1) \]

\[ = V_0 \sum_{n=1}^{\infty} [b_n \cos \omega_n(t - t_1) + c_n \sin \omega_n(t - t_1)] \]

(6)

which is the square pulse of Eq. (3), but delayed by time \( t_1 \). In other words, we obtain a square pulse that “switches on” at the time \( t_1 \). Hence, the square-designing pulse to be sent at \( z = 0 \) is

\[ v_T(0, t; z_1) \]

\[ = V_0 \sum_{n=1}^{\infty} [b'_n \cos(\omega_n t + \psi_n) + c'_n \sin(\omega_n t + \psi_n)] \]

(7)

where

\[ b'_n = b_n e^{\alpha_n z_1}, \]

(8)

\[ c'_n = c_n e^{\alpha_n z_1}, \]

(9)

and

\[ \psi_n = \beta_n z_1 - \omega_n t_1. \]

(10)

2. Experiment

Obviously, it is not possible to generate a perfectly square pulse with infinite components. A square pulse as described by Eq (3) with \( l' = 1 \mu s \), period \( T = 2.5 \mu s \) and \( n = 1, 2, \ldots, 10 \) was generated instead by using an arbitrary waveform generator (Tektronix AWG2003), and was sent through a coaxial cable (RG174/U) of lengths \( l = 50 \text{ m}, 100 \text{ m} \) and \( 150 \text{ m} \). With a 50 \( \Omega \) terminating resistor connected to the receiving end, the distorted pulse at the receiving end was observed by the use of an oscilloscope (Tektronix TDS3034). Fig. 1 shows the computed distorted pulse of Eq (4) in comparison with the observed pulse at the distances \( z_1 = 25 \text{ cm}, 50 \text{ m}, 100 \text{ m} \) and \( 150 \text{ m} \).

The square-designing pulses given by Eq (7) with \( l' = 1 \mu s, T = 2.5 \mu s \) and \( n = 1, 2, \ldots, 10 \), with \( z_1 = 50 \text{ m}, 100 \text{ m} \) and \( 150 \text{ m} \) were generated and sent through the cable. Fig. 2 shows the square-designing pulse for the designated point \( z_1 = 150 \text{ m} \) observed at two different distances: 25 cm (dotted line) and 150 m (solid line) where it is expected to have been “distorted” to become the desired pulse.
Figure 1: Comparison of the computed distorted waveform of Eq.(4) with the experimentally observed waveform at the distances (a) $z_1 = 25$ cm, (b) $z_1 = 50$ m, (c) $z_1 = 100$ m and (d) $z_1 = 150$ m.

Figure 2: Square-designing pulse (for $z_1 = 150$ m) observed at two different positions: 25 cm (dotted line) and 150 m (solid line).

In Fig. 3(a), 3(b) and 3(c), the observed desired pulses (blue curves) are compared with the distorted square pulses (green curves) of Eq.(4) and with the computed desired pulse (red curves) of Eq.(6) for the distances $z_1 = 50$ m, 100 m and 150 m respectively.

3. Discussion and Conclusion

In all figures of Fig. 3, it is seen that the curve for the observed desired pulse lies between the distorted pulse and the computed (ideal) desired pulse. In the peak region, the observed desired pulse is below the ideal pulse, while in the valley, the observed desired pulse is above the ideal pulse. A visual inspection shows that we have achieved a reduction of about 2/3 in the distortions.
A significant part of the difference between the theoretical curve as given by Eq.(4) and the experimental results as shown in Fig. 1 may be due to the adoption of the formulae with errors of up to 10\% (for the inductance) for the line parameters in [1]. The experimental curves are below the theory curve at the plateau and above the theoretical curve at the valley. As the same trend is observed in Fig. 3, it is likely that all the results can be improved by the use of more accurate formulae for the line parameters.

Acknowledgement

This research is supported by NIE Research Fund No. RP18/00FSK.

References
