On Cognitive Radio Systems with Directional Antennas and Imperfect Spectrum Sensing

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Abstract—In this paper, we consider a cognitive radio system, consisting of a primary user (PU), a secondary user (SU) transmitter, and a SU receiver. The SUs are equipped with directional antennas. The SU transmitter first performs spectrum sensing (with errors) and then transmits data. We assume the SU and PU can coexist and the SU transmits at two power levels, according to the result of spectrum sensing (i.e., whether the spectrum is sensed idle or busy). We establish a lower bound on the ergodic capacity of the channel between SU transmitter and receiver, and study how spectrum sensing errors affect the bound. Furthermore, we explore the optimal SU transmit power levels and the optimal directions of SU transmit and receive antennas, such that the lower bound is maximized, subject to average transmit power and average interference power constraints. Through numerical simulations, we show that (compared with the case when the SUs use omni-directional antennas) directional antennas can significantly improve the lower bound in the presence of spectrum sensing errors, subject to the constraints.

Index Terms—capacity maximization, cognitive radio, directional antenna, imperfect spectrum sensing.

I. INTRODUCTION

The communication paradigm of cognitive radios can alleviate spectrum scarcity problem, via allowing an unlicensed (cognitive or secondary) user (SU) to access the under-utilized licensed bands opportunistically, in such a way that its imposed interference on the licensed (primary) users (PUs) does not exceed the maximum allowed interference power level [1]. There is a rich collection of elegant results on optimizing transmission strategies for opportunistic spectrum access of SUs, in the presence of a PU activities [2]–[8]. The majority of these works assume the SUs are equipped with omni-directional antennas [2]–[8] and they transmit data only when the spectrum is sensed idle. Another commonly adopted assumption is that the result of spectrum sensing is perfect [4] (i.e., when the SU senses the spectrum is (not) occupied by the PU and is busy (idle), the spectrum is truly busy (idle)). However, all spectrum sensing methods (e.g., matched filter detection, energy detection) are prone to errors, quantified in terms of false alarm and detection probabilities, due to several sources of uncertainties including noise.

In this work, we assume the SUs are equipped with directional antennas. The directional antennas can identify and enable transmission and reception across spatial domain [9]–[12] and further increase spectrum utilization, compared with omni-directional antennas. Also, we assume the SUs and PU can coexist and the SU can adapt its transmission power, according to the result of spectrum sensing (i.e., whether the spectrum is sensed idle or busy). We study how spectrum sensing errors affect the ergodic capacity of the channel between SU transmitter and receiver. Furthermore, we explore the optimal SU transmit power and optimal orientations of SU transmit and receive antennas, such that the channel capacity lower bound is maximized, subject to average transmit power and average interference power constraints. To the best of our knowledge, this is the first work that combines the notions of directional antennas and imperfect spectrum sensing for cognitive radio systems.

II. SYSTEM MODEL

Fig. 1a depicts the cognitive radio system under consideration. The system consists of a PU, a SU transmitter (SUtx) and a SU receiver (SUrx). Suppose the SUs are equipped with steerable directional antennas and the main-lobes of SUtx and SUrx in their local coordination are centered on the angles $\phi_t$ and $\phi_r$, respectively. The directions of PU and SUtx with respect to SUtx in azimuth plane are denoted by $\theta_p$ and $\theta$, receptively. The direction of PU with respect to SUrx is denoted by $\theta'_p$. We assume the locations of PU and SUs are known and hence $\theta_p$, $\theta$ and $\theta'_p$ are known. The angles $\phi_t$ and $\phi_r$, however, are unknown and will be optimized.

A. Spectrum Sensing

We formulate the spectrum sensing problem at the SUtx as a binary hypothesis testing problem, where $H_1$ denotes that the spectrum is occupied by the PU and thus is truly busy, and $H_0$ indicates the spectrum is not occupied by the PU and hence is truly idle. Let $\pi_1 = \Pr\{H_1\}$ and

\[
\pi_0 = \Pr\{H_0\} = 1 - \pi_1.
\]
\( \pi_0 = \Pr\{H_0\} \), respectively, represent the probabilities that the spectrum is truly busy and truly idle. Let \( H_1 \) and \( H_0 \), respectively, denote that the result of spectrum sensing is busy and idle. The accuracy and reliability of any spectrum sensing method can be characterized in terms of false alarm and detection probabilities, defined as \( P_f = \Pr\{H_1|H_0\} \) and \( P_d = \Pr\{H_1|H_1\} \). Clearly, \( \Pr\{H_0|H_1\} = 1 - P_d \) and \( \Pr\{H_0|H_0\} = 1 - P_f \). Let \( \hat{\pi}_1 = \Pr\{H_1\} \) and \( \hat{\pi}_0 = \Pr\{H_0\} \), respectively, show the probabilities that the spectrum is sensed busy and idle. It is easy to verify \( \hat{\pi}_1 = \pi_0 P_f + \pi_1 P_d \) and \( \hat{\pi}_0 = \pi_0(1 - P_f) + \pi_1(1 - P_d) \). In this work, we assume \( \pi_0, P_f, P_d \) are known. Also, when the spectrum is truly busy, the PU transmission power level is \( \sigma^2_p \), although SU \( rx \) is unaware of this value. Transmit power level of SU \( rx \) depends on the result of spectrum sensing. When the spectrum is sensed idle and busy, SU \( rx \) uses transmit power \( P(0) \) and \( P(1) \), respectively.

**B. Data Communication Channel**

Let \( s_c[m] \) denote the discrete-time symbol transmitted by SU \( rx \) and \( r[m] \) represent the corresponding discrete-time signal received by SU \( rx \). We assume a block transmission/reception model where the SU \( rx \) transmit and receive several consecutive blocks of \( M \) symbols. We assume that during each block we have the following relationship

\[
r[m] = h_{s,s_r} \sqrt{G(\theta, \phi_t, \phi_r)} s_c[m] + n[m], \quad \text{for } m = 1, \ldots, M
\]

where \( G(\theta, \phi_t, \phi_r) = A(\phi_t - \theta)A(\phi_r - \pi - \theta) \) is the product of SU \( rx \) and SU \( tx \) antennas’ gain. The term \( h_{s,s_r} \) is the fading coefficient between SU \( tx \) and SU \( rx \). The term \( n[m] \) is the additive noise at SU \( rx \) and is modeled as Gaussian \( n[m] \sim N(0, \sigma^2_n) \). The transmitted symbols \( s_c[m] \) are digitally modulated signals and have the average power \( P(0) \) or \( P(1) \) when the spectrum is sensed idle or busy, respectively. We model the antenna gain \( A(\phi) \) as a function of direction \( \phi \) as \( A(\phi) = A_1 + A_0 \exp\left( -B \left( \frac{\phi}{\phi_0} \right)^2 \right) \) [11] where \( \phi_{\text{dB}} \) is the 3dB beam-width of antenna, \( B = \ln(2) \), \( A_1 \) and \( A_0 \) are two constant parameters and \( A_1 \) is the minimum antenna gain. This model is an approximation of a real antenna pattern used in [11], [12]. In Fig. 1b the antenna pattern is depicted for \( A_1 = 1, A_0 = 0 \) and \( \phi_{\text{dB}} = 30^\circ, 45^\circ \). Let \( d_{ps,t}, d_{ps,r}, d_{s,s_r} \), be the distances between PU and SU \( tx \), PU and SU \( rx \), and SU \( tx \) and SU \( rx \), respectively. All the fading coefficients include path loss and have Rayleigh distribution with variance \( \sigma^2_n = \left( \frac{d}{d_0} \right)^\nu \), where \( d_0 \) is the reference distance, \( d \) is the distance between users (SU or PU), and \( \nu \) is the path loss exponent.

**III. ERGODIC CAPACITY MAXIMIZATION**

We first characterize the ergodic capacity of the channel between SU \( tx \) and SU \( rx \), incorporating the fact that the result of spectrum sensing is imperfect, in terms of four optimization parameters: the antenna angles \( \phi_t, \phi_r \) and transmit power levels \( P(0) \) and \( P(1) \). Next, we study optimal \( \phi_t, \phi_r, P(0) \) and \( P(1) \), such that the channel capacity is maximized, subject to average transmit power constraint and average interference power constraint. For the clairvoyant scenario when spectrum sensing is perfect the maximum rate that the channel can support is \( C = E\{c_{0,0} + c_{1,1}\} \), where

\[
c_{0,0} = \log_2 \left( 1 + \frac{|h_{s,s_r}|^2 G(\theta, \phi_r) P(0)}{\sigma^2_n} \right),
\]

\[
c_{1,1} = \log_2 \left( 1 + \frac{|h_{s,s_r}|^2 G(\theta, \phi_r, \phi_r) P(1)}{\sigma^2_n + \sigma^2_p |h_{ps,t}|^2 A(\phi_r - \theta_p)} \right)
\]

\( E\{\cdot\} \) is the expectation operator and the expectation is taken with respect to random fading coefficients. The terms \( c_{0,0} \) and \( c_{1,1} \) are channel capacities when the spectrum is idle and busy, respectively and \( h_{ps,t} \) is the fading coefficient from PU to SU \( rx \). The term \( \sigma^2_p |h_{ps,t}|^2 A(\phi_r - \theta_p) \) in (2) captures the interference on SU \( rx \) due to PU activities. However, when spectrum sensing is imperfect, depending on the true status of the PU and the spectrum sensing result, two extra terms \( c_{0,1} \) and \( c_{1,0} \) would be added to the capacity expression where

\[
c_{0,1} = \log_2 \left( 1 + \frac{|h_{s,s_r}|^2 G(\theta, \phi_r, \phi_r) P(1)}{\sigma^2_n} \right),
\]

\[
c_{1,0} = \log_2 \left( 1 + \frac{|h_{s,s_r}|^2 G(\theta, \phi_r, \phi_r) P(1)}{\sigma^2_n + \sigma^2_p |h_{ps,t}|^2 A(\phi_r - \theta_p)} \right)
\]

Let \( \bar{I}_{av} \) denote the maximum allowed interference power level. To satisfy the average interference power constraint, we have

\[
E\left\{ \left| \hat{\pi}_0 P(0) + \beta_1 P(1) \right| \right\} |h_{ps,t}|^2 A(\phi_r - \theta_p) \leq \bar{I}_{av}.
\]

Let \( P_{av} \) indicate the maximum average transmit power of SU \( tx \). To satisfy the average transmit power constraint, we find

\[
\hat{\pi}_0 P(0) + \hat{\pi}_1 P(1) \leq P_{av}.
\]
problem $|h_{i,j}|^2$ has an exponential distribution with parameter $\lambda_{i,j} = \frac{(1 - \pi)}{\sigma h_{i,j}}$, where $i$ and $j$ can be $SU_{tx}$, $SU_{tx}$ or $SU_{pt}$. Thus, we can write $\mathbb{E}\{\ln (1 + |h_{i,j}|^2 G(z_i, \phi))\} = -e^{x^2/2} E\left[-\frac{a^2}{\pi^2 \sigma^2} e^{-ax} G(z_i, \phi)\right]$, where $a = \lambda_{i,j}$. Since the analytical maximization of the capacity is infeasible, instead, we establish a lower bound on the capacity, denoted as $C^{LB}$, and maximize $C^{LB}$. Using the inequality $\frac{1}{2} \ln (1 + \frac{\pi}{2}) < -e^x \ln (1 + \frac{1}{2})$ for $x > 0$ [14], we can write $\mathbb{E}\{\ln (1 + |h_{i,j}|^2 G(z_i, \phi))\} > \frac{1}{2} \ln (1 + \frac{2a^2}{\pi^2 \sigma^2})$ and obtain

$$C^{LB} = \frac{\bar{\pi}}{2} \log_2 \left(1 + \frac{2a^2 P^{(0)}}{\sigma^2}\right) + \frac{\pi}{2} \log_2 \left(1 + \frac{2a^2 P^{(1)}}{\sigma^2}\right). \quad (9)$$

In the following, we address the maximization of $C^{LB}$ with respect to four optimization parameters $\phi_0, \phi_1, P^{(0)}, P^{(1)}$, subject to two constraints in (5) and (6). Let $\phi^{opt}_P, \phi^{opt}_T, P^{opt}_0, P^{opt}_1$ be the optimal solutions. This optimization problem is convex with respect to $\phi_0, P^{(0)}, P^{(1)}$, but not with respect to $\phi_1$. Since $\phi_0$ lies in interval $[0,2\pi]$, $\phi^{opt}_P$ can be obtained using one-dimensional exhaustive search, i.e., we can consider an initial value for $\phi_0$ and solve the problem with respect to $\phi_1$. Then, we find the value of $\phi_0$ which maximizes the $C^{LB}$ [15]. Given $\phi_0$, the constrained maximization of $C^{LB}$ with respect to $P^{(0)}, P^{(1)}$, $P^{(1)}$ can be solved using the Lagrange multiplier method. By applying the Karush-Kuhn-Tucker (KKT) conditions, the optimum value for $\phi_0$ can be obtained as $\phi^{opt}_P = \pi + \theta$. We define

$$b_0 = A(\phi_0 - \theta_p) \partial_0 / \lambda_{st}, \quad b_1 = A(\phi_0 - \theta_p) \beta_1 / \lambda_{st}, \quad \Sigma = b_0 \tilde{P}_0 + b_1 \tilde{P}_1, \quad Y = b_0 \bar{P}_0 + b_1 \bar{P}_1,$$

$$\Psi = -b_0 \bar{P}_0 + \bar{P}_1, \quad \bar{P}_0 = \tilde{P}_0 - \tilde{P}_1.$$

The parameter $\Sigma$ can be simplified as $\Sigma = \frac{\tilde{P}_0}{\lambda_{st}} (P_d - P_f)A(\phi_0 - \theta_p)$. Assuming $P_d > P_f$, we conclude that $\Sigma > 0$. Solving the KKT conditions with regard to $P^{(0)}$ and $P^{(1)}$, we obtain the optimal transmit power under $\bar{H}_0$ and $H_1$.

$$P^{(0)} = \begin{cases}
\frac{\bar{P}_0}{\bar{P}_0}, & \Delta_2 < 0, \quad \Pi \geq 0, & \text{[case I]} \\
\frac{\bar{P}_0}{P_f}, & \Delta_0 \leq 0, & \text{[case II]} \\
\frac{\Delta_2}{2a^2 P^{(0)}}, & \Delta_3 < 0, \Delta_2 > 0, & \text{[case III]} \\
\Delta_3 > 0, \Delta_0 > 0, \quad \Pi < 0, \quad \Psi < 0 & \text{[case IV]} (10)
\end{cases}$$

where $\Delta_0 = (b_0 + b_1) \tilde{P}_0 - \bar{P}_0, \Delta_1 = 2a \tilde{P}_0 \bar{P}_0 \sigma^2 \Sigma$ and

$$\begin{align*}
\Delta_2 &= 2a \pi_1 \tilde{P}_{av} - \sigma^2 \Sigma \\
\Delta_3 &= \sigma^2 \Sigma^2 + 2a \bar{T} \tilde{P}_{av} - 2a \bar{P}_{av} b_0 b_1.
\end{align*}$$

In order to reduce the computational complexity of one-dimensional exhaustive search for finding $\phi^{opt}_P$ in the interval $[0,2\pi]$, in the following we find a narrower interval to which $\phi^{opt}_P$ belongs to, i.e., we find $\phi^{opt}_P$ such that $\phi^{opt}_P \in [\phi^1, \phi^2]$. The maximum imposed interference on PU would occur when $SU_{tx}$ always transmits data with maximum allowable transmit power $P^{(0)} = P^{(1)} = \bar{P}_0$ without considering the spectrum sensing result. In this case, considering (5) we obtain $A(\phi_0 - \theta_p) \leq \frac{\lambda_{st} \bar{P}_0}{\pi}$. We define $Z = (\lambda_{st} \bar{P}_0) / (\pi a_0 A_0) - A_0 / \lambda_{st}$. From (5) we have $\exp(-B(\frac{1}{\lambda_{st} \bar{P}_0})) \leq Z$. If $Z > 1$, it means that PU can tolerate an interference power that is larger than the interference power imposed by $SU_{tx}$ and (5) holds true for every value of $\phi_0$ and it is obvious that $\phi^{opt}_P = \theta$ maximizes $C^{LB}$. For $0 < Z \leq 1$, we define $\psi_p = \phi^{opt}_P \sqrt{\frac{1}{Z}} \ln (Z)$ and consider two cases. When $|\theta_p - \theta| > \psi_p$, $\phi^{opt}_P$ has to lie outside the shaded area shown in Fig. 2a. The unshaded area in Fig. 2a includes $\phi^{opt}_P \in [\phi^1, \phi^2]$. Given $\phi^{opt}_P$, the constrained maximization of $C^{LB}$ with respect to $P^{(0)}, P^{(1)}$, $P^{(1)}$ can be solved using the Lagrange multiplier method. By applying the Karush-Kuhn-Tucker (KKT) conditions, the optimum value for $\phi_0$ can be obtained as $\phi^{opt}_P = \pi + \theta$. We define

$$b_0 = A(\phi_0 - \theta_p) \beta_0 / \lambda_{st}, \quad b_1 = A(\phi_0 - \theta_p) \beta_1 / \lambda_{st}, \quad \Sigma = b_0 \tilde{P}_0 + b_1 \tilde{P}_1,$$

$$\Psi = -b_0 \bar{P}_0 + \bar{P}_1, \quad \bar{P}_0 = \tilde{P}_0 - \tilde{P}_1.$$

The parameter $\Sigma$ can be simplified as $\Sigma = \frac{\tilde{P}_0}{\lambda_{st}} (P_d - P_f)A(\phi_0 - \theta_p)$. Assuming $P_d > P_f$, we conclude that $\Sigma > 0$. Solving the KKT conditions with regard to $P^{(0)}$ and $P^{(1)}$, we obtain the optimal transmit power under $\bar{H}_0$ and $H_1$.

$$P^{(0)} = \begin{cases}
0, & \Delta_2 < 0, \quad \Pi \geq 0, & \text{[case I]} \\
\frac{\bar{P}_0}{\bar{P}_0}, & \Delta_0 \leq 0, & \text{[case II]} \\
\frac{\Delta_2}{2a^2 P^{(0)}}, & \Delta_3 < 0, \Delta_2 > 0, & \text{[case III]} \\
\Delta_3 > 0, \Delta_0 > 0, \quad \Pi < 0, \quad \Psi < 0 & \text{[case IV]} (10)
\end{cases}$$

1) $\phi^{opt}_P = \pi + \theta$.
2) Calculate $Z$.
3) Calculate the interval which contains $\phi^{opt}_P$.
   • Case a. If $Z > 1$ or if $Z < 1$ and $|\theta_p - \theta| > \psi_p$, then $\phi^{opt}_P = \theta$, i.e., no further optimization over $\phi_0$ is needed.
   • Case b. If $0 < Z < 1$ and $|\theta_p - \theta| < \psi_p$, then $\phi^{opt}_P$ lies in the interval mentioned in (12).
   • Case c. If $Z \leq 0$, then $\phi^{opt}_P$ lies in the interval $[0, 2\pi]$.
4) For case a, calculate $P^{(0)}, P^{(1)}$ using (10) and (11). For cases b and c, given a $\phi_0$ value within the obtained interval, calculate $P^{(0)}, P^{(1)}$ using (10) and (11).
5) For cases b and c, substitute $P^{(0)}, P^{(1)}, \phi_0, \phi^{opt}_P$ in (9), (10).
IV. NUMERICAL RESULTS AND CONCLUSION

Using Matlab simulations, we illustrate how directional antennas can improve the channel capacity when considering imperfect spectrum sensing. Assume $\sigma^2 = 1$, $\phi_{\text{3DB}} = 45^\circ$, $A_0 = 9$, $A_1 = 1$ and $\pi_1 = 0.3$. Let $\nu = 2$, $d_{p_{tx}} = 4$ m and $d_{s_{SU}} = 2$ m. Suppose $C^\text{Disc}$ denote $C^\text{LB}$ in (9), evaluated at the optimal solutions $\phi_{\text{opt}}^\text{tx}$, $\phi_{\text{opt}}^\text{SU}$, $P_{\text{opt}}^{(0)}$ and $P_{\text{opt}}^{(1)}$. We compare $C^\text{opt}$ with the lower bound on the channel capacity when SUrx and SUtx have omni-directional antennas with the antenna gain $A(\phi) = 10$ for all $\phi$, and only transmit powers $P^{(0)}$ and $P^{(1)}$ are optimized subject to the transmit power and interference constraints, which we denote as $C^\text{Omni}_{\text{opt}}$. Furthermore, to quantify the advantage of optimizing the angles of SUrx and SUtx directional antennas, we compare $C^\text{Disc}$ with $C^\text{LB}$ in (9), evaluated at $\phi = \theta$, $\phi = \pi + \theta$ (the antennas of SUtx and SUrx are exactly pointed at each other), $P^{(0)}$ and $P^{(1)}$ obtained from (10) and (11), which we denote as $C^\text{LOS}_{\text{opt}}$. For fair comparisons, we consider a fixed spectrum sensing method with $P_d = 0.9$ and $P_f = 0.1$. Note that $P^{(0)}_{\text{opt}}$ and $P^{(1)}_{\text{opt}}$ are constant for all $\theta$ when SU use omni-directional antennas and $C^\text{Omni}_{\text{opt}}$ is independent of $\theta$.

We define three capacity ratios $\Gamma_{\text{D2O}} = C^\text{Disc}_{\text{opt}} / C^\text{Omni}_{\text{opt}}$, $\Gamma_{\text{L2O}} = C^\text{LOS}_{\text{opt}} / C^\text{Omni}_{\text{opt}}$ and $\Gamma_{\text{D2L}} = C^\text{Disc}_{\text{opt}} / C^\text{LOS}_{\text{opt}}$. Fig. 4a plots $\Gamma_{\text{D2O}}$ and $\Gamma_{\text{L2O}}$ versus $\theta$ when $\theta_p = 0^\circ$, $P_{av} = 15$ dB and $I_{av} = 0$ dB for $\phi_{\text{3DB}} = 25^\circ$, $45^\circ$. It can be seen that when $\theta = 0$, $C^\text{Disc}_{\text{opt}} \approx C^\text{LOS}_{\text{opt}}$ and as $|\theta - \theta_p|$ increases, $C^\text{Disc}_{\text{opt}} > C^\text{LOS}_{\text{opt}}$. We observe that $C^\text{Disc}_{\text{opt}} > C^\text{LOS}_{\text{opt}}$ for $|\theta - \theta_p| < 135^\circ$ and for $|\theta - \theta_p| > 135^\circ$, $C^\text{Disc}_{\text{opt}} \approx C^\text{LOS}_{\text{opt}}$ when $\phi_{\text{3DB}} = 45^\circ$. Since $C^\text{Omni}_{\text{opt}}$ is independent of $\theta$, by comparing $\Gamma_{\text{D2O}}$ to $\Gamma_{\text{L2O}}$ in Fig. 4a, we can see that the trend of the curves in two figures are different. In fact, the shape of the curves for $\Gamma_{\text{D2O}}$ is similar to letter V. However, this shape is similar to letter U for $\Gamma_{\text{L2O}}$. It means that $\Gamma_{\text{D2O}}$ increases sharper (has a larger slope) with respect to $\theta$, compared with $\Gamma_{\text{L2O}}$. This effect is also shown in Fig. 4b where the performance gain of $C^\text{Disc}_{\text{opt}}$ against $C^\text{LOS}_{\text{opt}}$ is plotted. Also, in Fig. 4a we see that for a fixed value of $\theta$, as the beam-width decreases, the capacity gain increases. In other words, as half power beam-width decreases, the directional antenna can cancel more interference power imposed on or by the PU. Thus, the optimal capacity can reach to its maximum value for smaller $|\theta - \theta_p|$.

In summary, we considered a cognitive radio system, where the SU devices are equipped with directional antennas and spectrum sensing is imperfect. We explored the optimal SU transmit power levels and the optimal directions of SU antennas, such that the channel capacity lower bound is maximized, subject to average transmit power and average interference power constraints. Through simulations, we showed that directional antennas significantly enhance the lower bound.

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