CALIBRATION OF THE ATTENUATION-RAIN RATE POWER-LAW PARAMETERS USING MEASUREMENTS FROM COMMERCIAL MICROWAVE NETWORKS

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Abstract—A common way to describe the relation between rain-rate R \( [\text{mm/h}] \) and attenuation A \( [\text{dB}] \) in radio signal is the Power-Law \( A = aR^b \), where \( a \) and \( b \) are the Power-Law parameters, which depend on the radio signal (frequency, polarization) and on some properties of the specific environmental conditions. These parameters are usually set off-line using special purpose equipment and are used from existing tables. However, such tables provide averaged, approximated values to the Power-Law parameters. Using these values for local network design and/or for rainfall estimation can cause inaccuracies. In this paper we propose a new method for calibrating the power law parameters locally, in almost real time, using standard equipment - that is, measurements from rain-gauges and from existing commercial microwave networks deployed in cellular backhauling systems. We suggest an estimation procedure and demonstrate its operation using real scenario in the south of Israel.

Index Terms—Power-Law, Microwave Networks, Precipitation Attenuation

I. INTRODUCTION

The relationship between the rain-rate and the induced attenuation of radio signals, known as the \( aR^b \) relationship, or, simply as the Power-Law, has been established decades ago, and can be traced back to the 1940’s [14]. Later, this relationship has been shown to express the expected signal loss due to rain-rate accurately [9], [13], [22]. And so, the Power-Law has been, and is still being used as an important tool for wireless network designers and operators. The International Telecommunication Union (ITU) summarized a recommendation regarding the relationship between a Microwave Link (ML) attenuation and the rain-rate in its path, based on the same Power-Law [16]. This ITU recommendation (numbered 838-3, currently updated in 2005), is often considered as the technical standard used by the majority of the network designers.

On the other hand, in 2006, a novel approach suggested to use the same Power-Law for rain monitoring, based on the available attenuation measurements logged from the backhaul infrastructure of Commercial Microwave Networks (CMNs) [20]. The CMNs operators (i.e., the cellular providers) tend to log the MLs attenuation for their own monitoring and maintenance needs. Having access to the logged attenuation information has yielded opportunistic tools, designed for environmental monitoring, without the need for dedicated hardware installation, or other costs ( [3], [7], [11], [18], [29], among many others).

Regardless of the motivation (communication network design or environmental monitoring), the use of the Power-Law is crucial. Historically, the Power-Law parameters have been established based on physical reasoning such as Mie scattering, different Drop Size Distribution (DSD) assumptions, temperature, and so on [13], [22]. Currently, the widely used values of the Power-Law parameters are documented by the ITU [16], and are presented as tables of experimentally acquired values, which is periodically updated. Since these tables ignore the fact that the Power-Law parameters also depend (to some extent) on the local climate properties, in addition with the fact that these tables are empirically built, and so in some cases, more than 25% difference in the values of the parameters between each revision can be seen, it is probable that the accuracy of the currently used values may be problematic at times.

Indeed, since the Power-Law parameters are considered to be time-invariant (per location, considering that the climate of that location does not change over time), if a set of instantaneous attenuation measurements from an ML and a set of the rain-rate intensities from the same location were to be given, accurate values of the \( a \) and \( b \) parameters could have been evaluated. However, instantaneous attenuation measurements are rarely available: Even-though current CMNs is vastly installed, and its hardware is capable of recording the attenuation information at high temporal resolution\(^1\), the cellular providers do not require such high sampling rate. So, in order to save both bandwidth and storage, the current widely used protocols save only the minimum and the maximum measured Received Signal Level (RSL) and the Transmitted Signal Level (TSL) for every 15-minute interval [19] (from which the minimum and the maximum attenuation (per 15-minute interval) can be directly derived).

In this paper, we suggest a novel rigorous method for estimation of the Power-Law parameters: We show that it is possible to calibrate the parameters values of the Power-Law, by using actual measurements logged by the CMNs, and standard Rain-Gauges (RGs). Since CMNs, as well as RGs,

\(^1\) For example, the current Ericsson™ hardware is capable to sample and record the ML channel attenuation every 10 seconds [27].
are widely deployed, this new approach poses a great potential. We demonstrate our findings by a specifically designed experiment, which uses actual attenuation data gathered by a cellular operator, and present interesting results.

The rest of the paper is organized as follows: In section II we present the theory and the methodology of our approach. Section III details a real-world demonstration of the suggested approach. Next, Section IV discusses sources of errors, and concludes this manuscript.

II. THEORY AND METHODOLOGY

The known Power-Law, which relates the instantaneous rain-rate $R_i$, with the induced microwave signal attenuation $A_i$, has been validated both by theory [13], and by empirical means [9], [22], and is being used by various communication network engineers, as recommended by the ITU [16]:

$$A_i = a R_i^b \cdot L \quad [dB]$$  \hspace{1cm} (1)

where $A_i$ (in [dB]) is the $i^{\text{th}}$ sample of the instantaneous attenuation, $R_i$ (in $[mm/h]$) is the instantaneous rain-rate, $L$ (in $[km]$) is the ML length, and $a, b$ are the Power-Law parameters which depend on the ML frequency and polarization DSD and the temperature [13], and can be found in the literature [9], [16]. However, as presented in the Introduction, their registered values vary, and may be location-specific [13].

A. The Power-Law for Minimum and Maximum Attenuation

Consider the attenuation measurement vector $\mathbf{A} = [A_1, A_2, A_3, \ldots, A_N]^T$ which entries are sampled at constant intervals. And, consider the rain-rate measurement vector $\mathbf{R} = [R_1, R_2, R_3, \ldots, R_N]$ which entries are also sampled at constant intervals (for simplicity, we assume for the rest of this section that the sampling-intervals of $\mathbf{A}$ and $\mathbf{R}$ are of the same duration. Thus, $N = \bar{N}$). In combination with (1), the empirical averaged rain-rate $R_{avg}$ throughout the observation period, can be presented by:

$$R_{avg} = \frac{1}{N} \sum_{i=1}^{N} R_i = \frac{1}{N(a \cdot L)} \sum_{i=1}^{N} A_i^{\frac{b}{a}} \quad [mm/h]$$  \hspace{1cm} (2)

Although eq. (2) is straight-forward, it is unusable in most cases involving commercial microwave measurements, simply because, the instantaneous measurement vector $\mathbf{A}$ usually cannot be derived (since no instantaneous RSL/TSL measurements are available). [19].

In order establish a relationship between the average rain-rate $R_{avg}$ and the available measurements produced by the cellular operators (which are either the minimum or the maximum RSL and TSL levels, from which the minimum and the maximum attenuation is derived), we suggest to use the statistical properties of the rain.

It has been shown that the rain-rate intensity distribution resembles the general shape of the gamma (or the exponential) distribution families, but is not strict. The rain-rate distribution can be modelled by various distribution families [6], [21]. However, in this paper, we consider the rain-rate as an independent and identically distributed (iid) stochastic process, which follows the exponential distribution: $R_i \equiv R(t_i); \hspace{0.5cm}$ s.t $f_{R_i}(r, t_i; \theta) = \frac{1}{\theta} e^{-r(t_i)/\theta}$, where $t_i$ represents the $i^{\text{th}}$ time-index, and $\theta$ is the exponential PDF parameter.

And, under very mild regulatory conditions, (2) satisfies:

$$R_{avg} = \bar{E}[R_i] \rightarrow \bar{E}[R_i] = \theta \quad \frac{[mm]}{h}$$  \hspace{1cm} (3)

where $N$ is the number of the available samples, and $E[\cdot]$ is the expected value operator.

Next, consider the situation where from each group of $K \geq 2$ instantaneous attenuation observations, only the minimum or the maximum observed values are given (Thus, from $N$ original observation, $M = N/K$ minimum or maximum samples are available). For a specific minimum (or maximum) attenuation sample $A_{i_{min}}$ (or $A_{i_{max}}$), the Power-Law (1) is valid, and so, the average minimum rain-rate $R_{avg}^{\text{min}}$ can be calculated similarly to $R_{avg}$ (2):

$$R_{avg}^{\text{min}} = \frac{\sum_{i=1}^{M} (A_{i_{min}})^{\frac{b}{a}}}{M(a \cdot L)^{\frac{b}{a}}} \quad [mm/h]$$  \hspace{1cm} (4)

the same eq. (4) stands for the average maximum rain-rate $R_{avg}^{\text{max}}$ by using $A_{i_{max}}$.

B. $R_{i_{min}}$ and $R_{i_{max}}$ Statistics

By using the Extreme Value Theory (EVT), and under the assumptions made regarding the rain-rate process, it can be shown that both the minimum observed rain-rate $R_{i_{min}}$ and the maximum observed rain-rate $R_{i_{max}}$ follow the PDFs [12]:

$$f_{R_{i_{min}}}(r_{i_{min}}; \theta, K) = \frac{K}{\theta} e^{-r_{i_{min}}/\theta}$$  \hspace{1cm} (5a)

$$f_{R_{i_{max}}}(r_{i_{max}}; \theta, K) = \frac{K}{\theta} \left(1 - e^{-r_{i_{max}}/\theta}\right)^{K-1} e^{-r_{i_{max}}/\theta}$$  \hspace{1cm} (5b)

where $\theta$ is the same parameter of the original $R_i$ exponential distribution, and $K$ is the number of instantaneous observations from which the minimum or the maximum values have been taken.

From (5a), the expected value for $R_{i_{min}}$ can be found directly (since $f R_{i_{min}}(\cdot; \theta, K)$ is actually an exponential distribution with the parameter $\theta/K$):

$$E[R_{i_{min}}] = \frac{\theta}{K} \quad \frac{[mm]}{h}$$  \hspace{1cm} (6)

The expected value for $R_{i_{max}}$, on the other hand, is combersome, as it can only be represented as a series: $E[R_{i_{max}}] = \sum_{n=1}^{K} (\theta/n)$ [12]. However, by implementing Euler’s asymptotic approximation for harmonic series, the expression is simplified to:

$$E[R_{i_{max}}] \approx \theta \cdot (\ln(K) + \gamma) \quad \frac{[mm]}{h}$$  \hspace{1cm} (7)

where $\gamma$ is Euler’s constant, which equals to $\gamma = 0.57722$. And, as shown in [12], this approximation is accurate: For instance, for $K = 10$, the difference between the actual and

$^2$Although the rain-rate may not be iid for short sampling intervals [17], it has been shown that the rain-rate correlation diminishes quickly [10], [28]. Furthermore, it has been proved that the behavior of the extremes sensitivity to dependency is low [5]. So, the iid approximation can be justified in this case.
the approximated values of $E[R_{max}^i]$ is less than 1.7%. For $K = 90$, the difference drops to 0.11%.

C. Calibration of $a$ and $b$

Using the expressions of the expected values of $R_{min}^i$ (6) and $R_{max}^i$ (7), and assuming that $E[R_{min}^i]$ and $E[R_{max}^i]$ (4) are (asymptotically) consistent, the following equations can be derived:

$$E[R_{avg}^i] = \frac{\sum_{i=1}^{M} (A_{i}^{min})^\frac{1}{a}}{M(a \cdot L)^\frac{1}{a}} \approx \frac{\theta}{K} \left[ \frac{mm}{h} \right] \quad (8a)$$

$$E[R_{avg}^i] = \frac{\sum_{i=1}^{M} (A_{i}^{max})^\frac{1}{b}}{M(a \cdot L)^\frac{1}{b}} \approx \theta [\ln (K) + \gamma] \left[ \frac{mm}{h} \right] \quad (8b)$$

which, in combination with (3) becomes:

$$E[R_i] \approx \frac{\sum_{i=1}^{M} (A_{i}^{min})^\frac{1}{a}}{M(a \cdot L)^\frac{1}{a}} \left[ \frac{mm}{h} \right] \quad (9a)$$

$$E[R_i] \approx \frac{\sum_{i=1}^{M} (A_{i}^{max})^\frac{1}{b}}{M(a \cdot L)^\frac{1}{b}} \left[ \frac{mm}{h} \right] \quad (9b)$$

Eq. (9) expresses the relationship between the instantaneous rain-rate expected value and the minimum (or maximum) attenuation values. It can be seen, that this relationship (9) can be presented as in (2), given that the $a, b$ parameters are:

$$R_{avg} \approx \frac{\sum_{i=1}^{M} (A_{i}^{min})^\frac{1}{a_{cal}}}{M(a_{cal} \cdot L)^\frac{1}{a_{cal}}} \left[ \frac{mm}{h} \right] \quad (10a)$$

$$R_{avg} \approx \frac{\sum_{i=1}^{M} (A_{i}^{max})^\frac{1}{b_{cal}}}{M(a_{cal} \cdot L)^\frac{1}{b_{cal}}} \left[ \frac{mm}{h} \right] \quad (10b)$$

where

$$a_{cal} = \begin{cases} \frac{a}{K} & \text{for } A_{i}^{min} \\ a \cdot (\ln (K) + \gamma)^b & \text{for } A_{i}^{max} \end{cases} \quad (11a)$$

$$b_{cal} = b \quad (11b)$$

And so, from the measurement vector of $A_{min}^i$ and/or $A_{max}^i$, in combination with the knowledge of the average rain-rate, which can be acquired using RGs (as well as via weather radar, etc.), the $a, b$ parameters of the Power-Law can be extracted.

Next, we will present a demonstration of such an estimation, using actual CMN measurements, and RGs observations.

III. DEMONSTRATION USING ACTUAL CMN MEASUREMENTS

In order to demonstrate our approach, we designed an experiment based on actual CMN backhaul ML measurements, as well as RGs data, in order to estimate the value of the Power-Law parameter $a$ from the available maximum attenuation values and RGs observations. The location that was chosen is in the semi-arid south of Israel. The specific location was chosen due to the available ML and nearby RGs.

The ML measurements were logged by the Israeli cellular provider Cellcom™. The RGs have been operated by the Israeli Meteorological Services (IMS). The minimum and the maximum RSL and TLS measurements in 15-minute intervals were recorded from a 16[km] commercial ML, connecting the Israeli city of Arad with a nearby village named Beit-Yatir. From the RSL and the TSL measurements, the maximum and the minimum attenuation levels were directly derived. The ML operates at a frequency of 18.6[GHz] with horizontal polarization. The typical values of the Power-Law $a = 0.077$ and $b = 1.074$ parameters were taken from the literature [16]. Since the ML is based on the current Ericsson™ hardware, the original RSL and TSL measurements are sampled at 10-second intervals, from which, the minimum and the maximum measured values are reported every 15 minutes [27]. Hence, the value of $K$ in this scenario is $K = 90$, which is the number of 10-second intervals within 15 minutes. The rain-rate data were recorded as the accumulated rainfall (in mm) for every 10-minute interval, from two available RGs. The ML and the RGs locations can be seen in Fig. 1.

From January 2013 until February 2015, 13 occasions in which continuous rain of more than 2.5 hours have been detected by the RGs. Overall, 135 hours of rainy periods, arranged in 54 sections of 2.5 hours were collected and analyzed. For each of the 54 periods of rain, the average maximum attenuation ($A_{max}^i$ : $j \in [1, 54]$) have been derived from the ML measurements, and, the average rain-rate in the powers of $b$ ($R_{avg}^i$ : $j \in [1, 54]$; $b = 1.074$ [16]) have been calculated based on the RGs observations:

$$A_{avgj}^{max} = \frac{\sum_{i=1}^{10} A_{i}^{max}}{10} \left[ dB \right] \quad (12a)$$

$$R_{avgj}^{1.074} = \left( \frac{2 \sum_{i=1}^{15} (R_i)^{1.074}}{10} \right) \left[ \frac{mm}{h} \right] \quad (12b)$$

where $A_{avgj}^{max}$ (in [dB]) is the maximum attenuation measured for the $j$th interval, and $R_i$ (in $[mm/h]$) is the average rain-rate observed by the two RGs for the $i$th interval. Note, that since the RGs sampling interval is 10-minute, $R_{avgj}^{1.074}$ of (12) is normalized by a multiplication of 6, in order to be represented in the units of $[mm/h]$.

Once the measurements have been collected and prepared, we simply solved the equations set, where the variable vector is $a_{cal} = [a_{cal1}, a_{cal2}, \ldots, a_{cal54}]^T$:

$$A_{avg}^{max} = a_{cal} \cdot R_{avg} \cdot L \quad (13)$$

where $A_{avg}^{max} = [A_{avg1}^{max}, A_{avg2}^{max}, \ldots, A_{avg54}^{max}]^T$ and $R_{avg}^{b} = [R_{avg1}^{b}, R_{avg2}^{b}, \ldots, R_{avg54}^{b}]^T$. $L$ was taken as the ML path.
length \((L = 16\text{km})\), and \(b\) was taken as 1.074. From acal, the value of \(a\) for each of the 54 sections was calculated: 
\[
a_i = a_{cal} \cdot ((\ln(90) + 0.57722)^{-b}; i \in [1, 54].
\]
The results are presented in Fig. 2.

![Fig. 2. Scatter plot of the Power-Law parameter \(a\), calculated for 54 periods of 2.5 hours of rain, starting at the date and time indicated. The mean value of \(a\) (mean\(a\) = 0.046) is drawn, as well as the range of the literary values of \(a\), based on [9] (lower bound) and [16] (upper bound). The standard deviation of \(a\) is \(\sigma = 0.028\), and the range of \(a \pm \sigma\) is also drawn (in orange).](image)

Inspecting Fig. 2, it can be seen that in general, the calculated value of \(a\) (\(a = 0.046 \pm 0.028\)) resembles the values presented in the literature. Indeed, the mean value of \(a\) is a somewhat smaller than the literary values, which range from \(0.05\) [9] to \(0.08\) [16]. But is nonetheless in the same order of magnitude. It is worth noting, that once we tried to divide the 135 hours of rain into 27 sections of 5 hours (instead of the 54 sections of 2.5 hours each), the accuracy of the estimate slightly increased, as the SD value decreased by \(21\%\) (\(a_{5h-based} = 0.041 \pm 0.023\)). This is to be expected, since longer period-duration increases the accuracy of the estimation (eq. (3)).

Further discussion regarding the results and possible sources of errors is presented next.

IV. DISCUSSION AND CONCLUSION

The calculation of \(a\), performed in the previous section, presents an interesting result. The resulted \(a = 0.046 \pm 0.028\) is close, but, somewhat lower than the recommended literary values [9], [16]. The difference between the resulted value and the literature one can be explained two-fold: First, the methodology presented in this manuscript may be affected by various sources of errors. Second, since the experiment was executed in southern Israel, where the climate differs from climates in which the \(a\) and \(b\) parameters are calibrated for [16], [22], it may be that the value of \(a\) found in the presented demonstration is better suited for this location than the literary values. We will now discuss these remarks in detail:

A. Sources of Errors

In actual measurements there are some specific features, neglected in this paper, which can cause systematic errors [15], [19]. The main cause for such errors can be directed to the quantization noise, and the Zero-Level (ZL) setting (i.e., the signal attenuation from sources other than rain):

1) Quantization Noise: Due to bandwidth and storage constraints, a quantizer is implemented on the ML measurements. The quantization resolution in our RSL measurements are \(\pm0.3\text{[dB]}\). In addition, an Automated Transmission Power Control (ATPC) usually steps in and compensate for the signal loss by a feedback loop, which increases (or decreases) the transmitted power accordingly [19], [27]. The ATPC transmitted power induced a quantization noise of \(\pm1\text{[dB]}\) to our TSL measurements, which further increases the effective quantization levels. The quantization noise has been shown to disturb estimation methods based on CMNs attenuation measurements [29]. Lastly, it is worth noting, that the RGs also have quantization of \(\pm0.6\text{[mm/h]}\), which may affect the estimation accuracy.

2) Zero-Level Selection: The ZL selection remains an open debate, but it is known to affect the resulting estimates [4], [11], [19]. Moreover, it has been suggested that the ZL may fluctuate throughout the storm duration [25]. In this manuscript, for simplicity reasons, we have treated the ZL as a constant, which is represented by the attenuation level during the dry period prior to the storm. This simplifying assumption may induce additional errors into the estimation process.

B. Climate and Environmental Effects

During this study, the ML and the two RGs which were available are located in the south of Israel, which is a semi-arid climate zone [1]. This Israeli semi-arid climate is characterized by strong and sudden events of intense rainfall, which is unique in nature, compared to other climate zones [26]. In addition, this location experiences the Virga phenomenon [24], which describes a decrease in the rain-rate intensity as a function of the distance below the clouds, due to evaporation [8]. The fact that the height difference between the two ML basestations is roughly 200 meters, in an area which is susceptible to the Virga effect, creates a unique and esoteric scenario. This uniqueness may explain the lower value of the calculated \(a\), compared to the literary values. Due to these properties, it may be that the value of \(a\) evaluated in this paper is better suited to the specific area in which the experiment took place.

C. Future Research

In addition to the open questions mentioned throughout this paper, we feel that two additional interesting questions should be studied more deeply in the future: First, note that we demonstrated the estimation of \(a\), whereas \(b\) was considered known. The reasoning behind this is two-folded: First, for frequencies range of 18-19GHz, the value of \(b\) is very close to 1, and is rather constant, compared to the value of \(a\) [9], [16]. Second, due to a possible parameters coupling between the two parameters, estimation of both parameters at the same time requires more advanced estimation methods, which should be further inspected. Furthermore, since the quantization noise has been shown to have destructive effects on the minimum attenuation levels (due to the same quantization level but much smaller attenuation values) [2], [23], using the minimum attenuation levels in addition to the maximum ones may be difficult, and should be inspected in future research.
REFERENCES


