STOCHASTIC ONLINE CONTROL FOR SMART-GRID POWERED MIMO DOWNTLINK TRANSMISSIONS

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ABSTRACT

An infinite time-horizon resource allocation problem is formulated to maximize the time-averaged multi-input multi-output (MIMO) downlink throughput, subject to a time-averaged energy cost budget. By using the advanced time decoupling technique, a novel stochastic subgradient based online control (SGOC) approach is developed for the resultant smart-grid powered communication system. It is analytically established that even without a-priori knowledge of the underlying random processes, the proposed online algorithm is capable of yielding a feasible and asymptotically optimal solution.

Index Terms—MIMO broadcast channels, smart grids, high-penetration renewables, stochastic optimization.

1. INTRODUCTION

Downlink communications from the base station (BS) to mobile users in wireless cellular systems is usually analyzed as a Gaussian broadcast (BC) channel in information-theoretic approaches. Shannon’s capacity for both single-input-single-output (SISO) and multi-input multi-output (MIMO) BC channels has been well documented [1–4], when the transmitters (here BSs) are powered by persistent energy sources of the conventional electricity grid. However, the current grid infrastructure is on the verge of a major paradigm shift, migrating from the aging grid to a “smart” one.

While integration of smart-grid technologies into resource allocation clearly holds the key to fully exploiting the potential of future downlink communications [5], only a few works are available in this direction. Leveraging limited smart-grid capabilities in simplified smart-grid models, recent works [5,6] addressed energy-efficient resource allocation for coordinated cellular downlink transmissions. Building on realistic smart-grid models, our recent works in [7,8] developed energy management to minimize the energy transaction cost subject to user quality-of-service (QoS) guarantees of coordinated cellular downlink settings. None of these works though touched on the impact of advanced smart-grid capabilities on the fundamentally achievable rate limits for the BC channels in cellular networks.

As MIMO techniques have been well adopted by wireless standards, we study here the optimal resource allocation for smart-grid powered MIMO downlink transmissions to approach the fundamentally achievable rate limits in future cellular networks. Specifically, we develop an online resource allocation approach, which dynamically makes instantaneous decisions without a-priori knowledge of any statistics of the underlying random channel, renewables, and electricity price processes. To this end, the intended task is formulated as an infinite horizon optimization problem aiming to maximize the time-averaged (weighted) downlink throughput subject to a time-averaged energy cost budget. Targeting a low-complexity solution, we adopt the relaxation techniques in [8,9] to decouple the decision variables across time. Leveraging the stochastic dual-subgradient method, we propose a novel online control algorithm. To analyze our scheme, we generalize the framework in [8,9] to characterize the two coupled “virtual” queues involved in our online control. We then establish analytically that the proposed algorithm can yield a feasible and asymptotically optimal strategy for the original problem.

The rest of the paper is organized as follows. The system models are described in Section 2. The proposed dynamic resource allocation scheme is developed and analyzed in Section 3. Numerical results are provided in Section 4, followed by the conclusions.

2. SYSTEM MODELING

Consider a MIMO BC downlink where a BS with $N_t$ antennas communicates to $K$ mobile users, each having $N_r$ antennas. Powered by a smart microgrid, the BS is equipped with one or more energy harvesting devices (solar panels and/or wind turbines), and can perform two-way energy trading with the main grid. In addition, the BS has a battery so that it can store part of the harvested energy for later use.

2.1. MIMO Downlink Channels

Consider an infinite scheduling horizon, indexed by the set $\mathcal{T} := \{0, 1, 2, \ldots \}$. Per slot $t \in \mathcal{T}$, let $H_{k,t} \in \mathbb{C}^{N_r \times N_t}$ denote the channel coefficient matrix from the BS to user $k = 1, \ldots, K$, and $\mathcal{H}_t := \{H_{1,t}, \ldots, H_{K,t}\}$. Let $x(t) \in \mathbb{C}^{N_t \times 1}$ denote the transmitted vector signal, which is the superposition of those transmitted to individual users: $x(t) = \sum_{k=1}^K x_k(t)$. The complex-baseband signal received at user $k$ is then

$$y_k(t) = H_{k,t}^\dagger x(t) + z_k(t)$$

where $z_k(t)$ is additive complex-Gaussian noise with zero mean and covariance matrix $I$ (the identity matrix of size $N_t$).

The MIMO BC capacity is known to be achievable by dirty paper coding [10]. For the codeword $x_k(t)$, the transmit covariance matrix of user $k$ is $S_{k,t} := \mathbb{E}[x_k(t)x_k^\dagger(t)]$. With $P_{t,k}$ denoting the transmit-power budget at the BS, it holds that $\sum_{k=1}^K \operatorname{tr}(S_{k,t}) \leq P_{t,k}$. With $r_k(t)$ denoting the achievable transmission rate for user $k$, $\pi(k)$ the $k$th user of a certain permutation $\pi$ of $\{1, 2, \ldots, K\}$, and $\cdot \mid \cdot$ the determinant operator, the rate $K$-tuple corresponding to per-
mutation $\pi$, is

$$R_\pi(P_{x,t}; H_t) = \bigcup_{\{r_1, \ldots, r_K\}} \{ (r_1, \ldots, r_K) : r_{\pi(k)} \leq \log \left| \frac{I + \sum_{u=1}^{K} H_{z(u,t)} H_{\pi(u,t)}}{I + \sum_{u=1}^{K} H_{z(u,t)} S_{\pi(u,t)} H_{\pi(u,t)}} \right|, \forall k \}$$

(2)

and the BC capacity region per slot is

$$C_{BC}(P_{x,t}; H_t) = C_0(\bigcup_{\pi} R_\pi(P_{x,t}; H_t))$$

(3)

where $C_0(\cdot)$ denotes the convex hull of the union over all permutations $\pi$ of $\{1, 2, \ldots, K\}$.

2.2. Smart Grid Operations

Let $E_t$ denote the BS energy harvested at the beginning of slot $t$. Further, let $C_0$ denote the initial energy, and $C_t$ the state of charge (SoC) in the battery at the beginning of slot $t$. With $C^{\text{max}}$ and $C^{\text{min}}$ bounding the capacity in the battery, we have $C^{\text{min}} \leq C_t \leq C^{\text{max}}$, $\forall t$. With $P_oh$ denoting the energy delivered to or drawn from the battery at slot $t$, the stored energy obeys $C_{t+1} = C_t + P_oh$, where the power (dis)charged is bounded by $P_o^{\text{min}} \leq P_oh \leq P_o^{\text{max}}$. Per slot $t$, the total BS energy consumption $P_{oh,t}$ includes the transmission-related power $P_{oh,t}$ and a constant power $P_c$ due to other components such as the data processor, and circuits; hence, $P_{oh,t} = P_c + P_{oh,t}$, where it is further assumed that $P_{oh,t} \leq P_c^\text{max}$.

When the renewable harvested energy is insufficient, the main grid can supply the needed $P_oh,t$ to the BS with an amount $[P_{oh,t} - (E_t + P_c)]^+$; with a two-way energy trading mechanism present, the BS can also sell its surplus energy $[P_{oh,t} - (E_t + P_c)]^-$ to the grid at a fair price in order to reduce operational costs, where $[a]^+ := \max\{a, 0\}$, and $[a]^− := \max\{-a, 0\}$. Suppose that the energy can be purchased from the grid at a price $\alpha_i \in [\alpha_0, \alpha_{\text{max}}]$, while the energy is sold to the grid at price $\beta_t \in [\beta_{\text{min}}, \beta_{\text{max}}]$, with $\alpha_i > \beta_t$ per slot $t$. Per slot $t$, the transaction cost for the BS is given by

$$G(P_{oh,t}, P_c,t) = \alpha_0 [P_{oh,t} - (E_t + P_c)]^+ - \beta_0 [P_{oh,t} - (E_t + P_c)]^-.$$

3. DYNAMIC RESOURCE ALLOCATION

Let $w_k$ denote the priority weight for user $k$, $S_i := \{S_{i,t}, \ldots, S_{i,K} \}$, and $G^{\text{max}}$ the maximum allowable power cost at the BS. Let $r^B(S_i)$ denote the achievable transmission rate for user $k$, and $r^P(S_i) := [r^P(I(S_i), \ldots, r^P_i(S_i))].$ Over the scheduling horizon $T$, the central controller at the BS optimizes the achievable per-user rates and battery charging power $[P_{oh,t}, P_c,t]$, in order to maximize the average (weighted) total throughput, subject to average energy cost constraint. For notational brevity, we introduce the auxiliary variable $P_{i,t} := P_{oh,i} + P_{c,i}$, and express the variables $P_{oh,i}$ terms of $\{P_i, P_{oh,i}\}$. In sum, we wish to solve

$$\max_{(S_t, C_t, P_{i,t})} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{k=1}^{K} w_k \cdot (r^B(S_i))$$

(4a)

subject to

$$0 \leq P_{oh,t} \leq P^{\text{max}}_o$$

(4b)

$$P^{\text{min}}_o \leq P_{oh,t} - P_{c,t} \leq P^{\text{max}}_c$$

(4c)

$$C_{t+1} = C_t + P_{oh,t} - P_{c,t}$$

(4d)

$$C^{\text{min}} \leq C_t \leq C^{\text{max}}$$

(4e)

$$r^B(S_i) \in C_{BC}(P_{i,t}; H_t), \forall t.$$

(4f)

3.1. Reformulation and Relaxation

With $\psi_1 := (\alpha_i - \beta_t)/2$ and $\psi_2 := (\alpha_i + \beta_t)/2$, it follows that $G(P) = \psi_1 |P_{oh,t}| + \psi_2 (P_{c,t})$. Since $\alpha_i > \beta_t > 0$, we have $\psi_1 > \psi_2 > 0$; hence, $G(P)$ is a convex function of $P_t$.

Now let us convexify the rate functions $r^P_i(S_i)$. By the uplink-downlink duality [11–13], the BC capacity region $C_{BC}(P_{x,t}; H_t)$ can be alternatively characterized by the capacity regions of a set of “dual” multi-access channels (MACs). In the dual MAC, the received signal is $y(t) = \sum_{k=1}^{K} H_{z(k,t)} x(k,t)$, where $x_k(t)$ is the signal transmitted by user $k$, and $z(t)$ is additive complex-Gaussian noise with zero mean and covariance matrix $I$. Let $Q_k := E[|x_k(t)|^2] \geq 0$ denote the transmit covariance matrix of user $k$, and let $p := [P_1, \ldots, P_K]$ collect the transmit-power budgets of all users. The uplink-downlink duality dictates that the BC capacity region (3) equals the union of the MAC capacity regions corresponding to all power vectors $p$ satisfying $\sum_{k=1}^{K} P_k \leq P_{oh,t}$, that is,

$$C_{BC}(p; H_t) := \bigcup_{p} \{ (r_1, \ldots, r_K) : r_{\pi(k)} \leq \log \left| \frac{I + \sum_{u=1}^{K} H_{z(u,t)} Q_{\pi(u,t)} H_{\pi(u,t)}}{I + \sum_{u=1}^{K} H_{z(u,t)} S_{\pi(u,t)} H_{\pi(u,t)}} \right|, \forall k \}$$

(5)

With $R_t(P_{x,t}) := \max_{p} r^P_i(S_i) \in C_{BC}(p; H_t)$, $\forall t$, $p = [P_1, \ldots, P_K]$ is a convex optimization problem.

Lemma 1 Function $R_t(P_{x,t})$ can be obtained by the optimal value of

$$\max_{Q_k \geq 0} \frac{1}{T} \sum_{t=0}^{T-1} \frac{1}{K} \sum_{k=1}^{K} \frac{1}{K} \sum_{u=1}^{K} (u(t)) \log \left| I + \sum_{u=1}^{K} H_{z(u,t)} Q_{\pi(u,t)} H_{\pi(u,t)} \right|$$

subject to

$$\sum_{u=1}^{K} (u(t)) Q_k = P_{x,t}$$

(6)

where $\pi(t)$ is the permutation of user indices $\{1, \ldots, K\}$ such that $w_{\pi(t)} \geq \cdots \geq w_{\pi(K)}$, and $w_{\pi(K+1)} = 0$. In addition, $R_t(P_{x,t})$ is a strictly concave and increasing function of $P_{x,t}$.

Using $R_t(P_{x,t})$, the optimal BC problem can be converted into the optimal sum-power allocation for an equivalent “point-to-point” link, as follows

$$R^* := \max_{(C_t, P_{i,t})} \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} [R_t(P_{x,t})],$$

(7)

subject to

$$0 \leq P_{oh,t} \leq P_{oh}^{\text{max}}$$

(4b)

$$P_{oh}^{\text{min}} \leq P_{oh,t} - P_{c,t} \leq P_{oh}^{\text{max}}$$

(4c)

$$C_{t+1} = C_t + P_{oh,t} - P_{c,t}$$

(4d)

$$C^{\text{min}} \leq C_t \leq C^{\text{max}}$$

(4e)

$$r^P_i(S_i) \in C_{BC}(P_{i,t}; H_t), \forall t.$$
\[
\max_{Q_k \geq 0, P_t \geq 0} \sum_{k=1}^{K} (w_{\pi(k)} - w_{\pi(k+1)}) \log |I + \sum_{u=1}^{k} H^f_{\pi(u),t} Q_{\pi(u)} H_{\pi(u),t}| + \lambda_2 \sum_{k=1}^{K} \text{tr}(Q_k) - \lambda_1 (j) G(P_t) \\
\text{s.t. } 0 \leq \text{tr}(Q_k) \leq P_{h_k}^{\max} - P_t, \quad P_{h_k}^{\min} \leq P_t - \sum_{k=1}^{K} \text{tr}(Q_k) \leq P_{h_k}^{\max}
\]

and likewise for \(E[G(P_t)], E[P_t], \) and \(E[P_{h_k}], \) where the expectation is taken over all sources of randomness. Now simply remove the variables \(C_t, \) and consider the following problem

\[
\hat{\mathcal{R}}^* := \max_{P_t, P_{h_k}} E[R_t(P_{x,t})] \\
\text{s.t. } E[G(P_t)] \leq G^{\max}, \quad E[P_t] = P + E[P_{x,t}]
\]

(9)

It can be shown that (9) is a relaxed version of (7). Specifically, any feasible solution of (7) also satisfies the constraints in (9) [15]. As a result, the optimal value of (9) is not less than that of (7); that is, \(\hat{\mathcal{R}}^* \geq R^*.\) Note that the time coupling constraint (4e) has been relaxed in problem (9), which then becomes easier to solve.

We next develop a stochastic dual subgradient solver for (9), which under proper initialization can provide an asymptotically optimal solution of the original resource allocation problem (4).

3.2. Dual Subgradient Approach

Let \(\mathcal{F}_t\) denote the set of \(\{P_t, P_{h_k}\}\) satisfying constraints (4c)-(4d) per slot, and \(\lambda := [\lambda_1, \lambda_2]\) collect the Lagrange multipliers associated with the two average constraints. With the convenient notation \(X_t := (P_t, P_{h_k})\) and \(X := (X_t, \forall t),\) the partial Lagrangian function of (9) is

\[
L(X, \lambda) := E[R_t(P_{x,t})] - \lambda_1 (E[G(P_t)] - G^{\max}) - \lambda_2 (E[P_t] - P - E[P_{x,t}])
\]

(10)

while the Lagrange dual function is given by \(D(\lambda) := \max_{X \in \mathcal{F}_t} L(X, \lambda),\) \(L(X, \lambda),\) and the dual problem of (9) is: \(\min_{\lambda_1, \lambda_2} D(\lambda).\)

For the dual problem, we can resort to a standard subgradient method to obtain \(X^*.\) This amounts to running the iterations

\[
\lambda_1 (j + 1) := \lambda_1 (j) - \mu (G^{\max} - E[G(P_t(j))]) + \lambda_1 (j) G(P_t) - \lambda_2 (j) (P_t - P - E[P_{x,t}])
\]

\[
\lambda_2 (j + 1) := \lambda_2 (j) - \mu (P_t + E[P_{x,t}(j)] - E[P_t])
\]

where \(j\) is the iteration index, and \(\mu > 0\) is an appropriate stepsize; while primal variables \(P_t(j)\) and \(P_{x,t}(j)\) are given by

\[
\{P_t(j), P_{x,t}(j)\} \in \arg \max_{(P_t, P_{x,t}) \in \mathcal{F}_t} \{R_t(P_{x,t}) - \lambda_1 (j) G(P_t) - \lambda_2 (j) (P_t - P - E[P_{x,t}])\}.
\]

(12)

By Lemma 1, the convex maximization problem (12) can be transformed into (13) at the top of the page, which can be efficiently solved by an off-the-shelf solver in polynomial time. With \(\{P_t(\lambda(j)), Q_{\pi}(\lambda(j)), \forall k\}\) denoting the optimal solution of (13), one can subsequently determine \(P_t(j) = P_t(\lambda(j)),\) and \(P_{x,t}(j) = \sum_{k=1}^{K} \text{tr}(Q_k(\lambda(j))).\)

3.3. Stochastic Subgradient Online Control

A challenge associated with the subgradient iterations (11) is computing \(E[P_t(j)], E[P_{x,t}(j)],\) and \(E[G(P_t(j))]\) per iterate. This amounts to performing (high-dimensional) integration over unknown joint distribution functions; or approximately, computing the corresponding time-averages over an infinite time horizon. To overcome this impractical requirement, we resort to a stochastic subgradient approach. Dropping \(E\) from (11), consider the iterations

\[
\lambda_1 (j + 1) = [\lambda_1 (j) - \mu (G^{\max} - G(P_t(\lambda(j))))] + \lambda_1 (j) G(P_t) - \lambda_2 (j) (P_t - P - E[P_{x,t}])
\]

\[
\lambda_2 (j + 1) = \lambda_2 (j) - \mu (P_t + E[P_{x,t}(\lambda(j)) - E[P_t])
\]

where \(\lambda_1^*, \lambda_2^*\) are stochastic estimates of those in (11), and \(P_t(\lambda^*), P_{x,t}(\lambda^*)\) are obtained by solving (12) with \(\lambda(j)\) replaced by \(\lambda^*\).

The update (14) is in fact an online approximation algorithm based on the instantaneous decisions \(\{P_t(\lambda^*), P_{x,t}(\lambda^*)\}\) per slot. Based on (14), we will develop next a stochastic subgradient based online control (SOGC) algorithm for the original problem (4). The algorithm is implemented at the BS as follows.

**SGOC:** Initialize with a proper \(\lambda^0 := \lambda_1^*{\lambda}_2^*.\) At every time slot \(t,\) observe \(\hat{\lambda}_1, \hat{\lambda}_2, \hat{H}_t, \hat{E}_t, \alpha_t, \beta_t,\) and then do:

**Energy management.** Obtain \(\{P_t(\lambda^*), P_{x,t}(\lambda^*)\}\) by solving (12). Perform energy transaction with the main grid; and (dis)charge the battery with the amount \(P_{h_t} = P_t(\lambda^*) - P_{x,t}(\lambda^*) - P_r.\)

**Broadcast schedule.** Given \(P_t(\lambda^*), P_{x,t}(\lambda^*)\) at the BS, solve the convex problem (6) to obtain the optimal “dual” MAC transmit-covariance matrices \(\{Q_{\pi}(P_{x,t}(\lambda^*)), \forall k\}\). Define for \(k = 1, \ldots, K,\)

\[
A_k := I + H_{\pi(k)} \left( \sum_{u=1}^{K} S_{\pi(u),t-1} H_{\pi(u),t}^\dagger \right) H_{\pi(k),t}^\dagger
\]

\[
B_k := I + \sum_{u=1}^{K} \left( \sum_{k=1}^{K} S_{\pi(u),t-1} H_{\pi(u),t}^\dagger \right) H_{\pi(k),t}^\dagger H_{\pi(u),t}
\]

and use them to find the covariance matrices

\[
S_{\pi(k),t} = B_k^{-\frac{1}{2}} F_k G_k A_k^{\dagger} Q_{\pi(k)}(P_{x,t}(\lambda^*)) A_k^{-\frac{1}{2}} G_k F_k^{\dagger} B_k^{-\frac{1}{2}}
\]

where \(F_k\) and \(G_k\) could be obtained via singular value decomposition of the effective channel \(H_{\pi(k)},\) given by \(B_k^{-\frac{1}{2}} H_{\pi(k)} A_k^{-\frac{1}{2}} = F_k G_k^{\dagger}\) with a square and diagonal matrix \(Z\) [12]. Perform MIMO broadcast with covariance matrix \(S_{\pi(k),t}\) per user \(k.\)

**Lagrange multiplier updates.** With \(P_t(\lambda^*), P_{x,t}(\lambda^*)\) available, update Lagrange multipliers \(\lambda^{j+1}\) via (14).

3.4. Performance Guarantees

Next, we will rigorously establish that the proposed algorithm asymptotically yields a feasible and optimal solution of (4) under proper initialization. To this end, we first assert the asymptotic optimality of the proposed SOGC algorithm in the following sense:†

**Lemma 2.** If \(\{H_t, E_t, \alpha_t, \beta_t\}\) are i.i.d. over slots, then the time-averaging throughput under the proposed SOGC algorithm satisfies

\[
R^* \geq \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} E[R_t(P_{x,t}(\lambda^*))] \geq R^* - \mu M
\]

where \(M := \frac{1}{2}(\max\{P_{h_k}^{\max} - P_{h_k}^{\min}\})^2 + (G^{\max})^2 + \max\{\alpha_{\max}\ (P_{h_k}^{\max} - P_{h_k}^{\min}), (\beta_{\max} - P_{h_k}^{\min})\}^2\).

†The proofs of all lemmas and the theorem are omitted due to limited space, but can be found in the journal version of this paper [15].
Lemma 2 asserts that the proposed SGOC algorithm converges to a region with optimality gap smaller than $\mu M$, which vanishes as the stepsize $\mu \to 0$. However, since the proposed algorithm is based on a solver for the relaxed (9), it is not guaranteed that the resultant dynamic control policy is a feasible one also for (7). In the sequel, we will establish that the SGOC in fact can yield a feasible policy for (7), when it is properly initialized.

Let $R'(P_{x,t})$ denote the left (or right) derivative of $R(P_{x,t})$. Define $R'(0) := \max\{R'(0), \forall t\}$, and assume $R'(0) < \infty$. Using the compact notation $\delta_{\lambda_1} := \max\{0, \alpha_{\max}(P_{x}^{\max} + P_{b}^{\max}) - C_{\max}\}$, we have established the following.

**Lemma 3** If the stepsize satisfies $\mu \geq \frac{\mu}{\delta_{\lambda_1}}$, where

$$\mu := \frac{\alpha_{\max}(C_{\max} - C_{\min} + \mu P_{b}^{\min} - \mu P_{b}^{\max} - \delta_{\lambda_1})}{\mu_{\min}} \tag{15}$$

the SGOC iterates satisfy $\lambda_1 \in [-\alpha_{\max}(R'(0) / \mu_{\min}) + \mu P_{b}^{\min}, \mu C_{\max} - \mu C_{\min} - \alpha_{\max}(R'(0) / \mu_{\min}) + \mu P_{b}^{\max}]$.

Note that $\lambda_1$ and $\lambda_2$ can be seen as two “virtual queues,” and the evolution of $\lambda_2$ in (14) in fact depends on the value of $\lambda_1$; in other words, the two “virtual queues” are coupled. This coupling of “virtual queues” complicates the analysis, and it is clearly different from [8,9], where the “virtual queues” evolve independently. Yet, by exploiting the revealed characteristics of our SGOC policy, we can first upper and lower bound $\lambda_1$. Then capitalizing on the specific coupling of the two “queues,” we further derive the stepsize lower bound $\mu$ to ensure the bounds in Lemma 3 for $\lambda_1$

We consider now the mapping between the real and virtual energy queues: $C_{t} = \frac{\lambda_1}{\mu_{\min}} + \alpha_{\max}(R'(0) / \mu_{\min}) + \mu P_{b}^{\min} + \mu_{\min} \delta_{\lambda_1} + C_{\min} - P_{b}^{\min}$

It can be readily inferred from Lemma 3 that $C_{\min} \leq C_t \leq C_{\max}$ holds, $\forall t$; hence, (4f) is always satisfied under the SGOC. Based on Lemmas 2 and 3, we arrive at the main result.

**Theorem 1** If we initialize with $\lambda_2 = \mu C_{0} - \mu C_{\min} + \mu P_{b}^{\min} - \alpha_{\max}(R'(0) / \mu_{\min}) + \mu_{\min} \delta_{\lambda_1}$, and select $\mu \geq \mu_{\min}$, then the proposed SGOC yields a feasible dynamic control for (7), which is asymptotically optimal in the sense that $\lim_{t \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[R(P_{x,t}(\lambda_1))] \geq \gamma - \mu M$.

Clearly, the minimum optimality gap between the SGOC and the offline scheduling is given by $\mu M$. The asymptotically optimal solution can be attained if we have very small power purchase prices $\alpha_{t}$, or, very large battery capacities $C_{\max}$, so that $\mu \to 0$. This makes sense intuitively because when the BS battery has a large capacity, the upper bound in (4f) is loose. In this case, with proper initialization, the SGOC using any $\mu$ will be feasible for (7), or, (4).

### 4. NUMERICAL RESULTS

The considered MIMO downlink has a BS with $N_{t} = 2$ antennas, communicating to $K = 10$ mobile users equipped with $N_{u} = 2$ antennas each. The system bandwidth is 1 MHz, and each entry of $H_{0,t}$ is a zero-mean complex-Gaussian random variable with unit variance. The maximum/minimum SoCs are set to $C_{\max} = 50$, $C_{\min} = 0$, while the (dis-)charging rates are set to $P_{b}^{\min} = -5$ and $P_{b}^{\max} = 5$ kW/hour. The energy purchase price $\alpha_{t}$ is uniformly distributed over $[0.1, 1]$. Our selling price is set as $\beta_{t} = \alpha_{t} \beta_0$ with $\beta_0 = 0.9$. The stepsize is chosen as $\mu = \frac{\mu}{\delta_{\lambda_1}}$. Two baseline schemes are introduced in this setup, where ALG 1 is a “greedy” scheme that maximizes the instantaneous throughput in (7) per slot.

### 5. CONCLUSIONS

Real-time resource allocation was developed for smart-grid powered MIMO downlinks. Relying on the stochastic subgradient method, a novel online algorithm was proposed to obtain a feasible and asymptotically optimal solution without knowing the distribution of the underlying stochastic processes. Our generalized performance analysis framework has fairly broad applicability for online control of wireless networks with coupled “real” or “virtual” queues.
6. REFERENCES


