ADAPTIVE MODIFICATION OF TRANSFORM COEFFICIENTS FOR IMAGE COMPRESSION

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ABSTRACT

For image compression purposes a general framework for modification of transform coefficients is proposed in this paper. To show the functionality of the method, we applied it to the contourlet transform. Unlike non-linear approximation (NLA) algorithms, the proposed algorithm causes the modification of the coefficients to be performed in a controlled manner. Different coefficients in different scales of the contourlet transforms have various roles in the reconstruction of the details of an image. The proposed algorithm adaptively modifies the coefficients based on the importance of the coefficient's scale. We rank the scales and their coefficients so that the coefficients with higher impact scales are modified within smaller bounds. Larger modifications are applied to coefficients of lesser importance. All of the modifications are performed with the goal of minimizing the entropy of the coefficients. The implementation results show that our algorithm produces better numerical results and better visual qualities, especially for images with fine and regular textures, at low bit-rates.

Index Terms—Image Compression, low bit-rate, contourlet, non-linear approximation

1. INTRODUCTION

We are witnessing that the number of images being transmitted in the internet or being communicated by wireless devices is growing. This has created an ever growing demand for more efficient and accurate compression algorithms. Lossy compression algorithms, which are, mostly based on transform methods [1], can be applied in applications where the exact reconstruction of the original image is not necessary. The wavelet transform (WT) has shown its high capability to compress natural images that have smooth regions with distinct boundaries. But WT is not very versatile when contours are encountered. Also textured images are not suitable for application of wavelet [2].Due to the mentioned shortcomings of the WT other transforms are suggested such as bandelet [2], curvelet [3], and contourlet [4].

The contourlet transform (CT), is a geometric transform which efficiently captures features such as contours and textures. Two main parts of the contourlet transform are Laplacian pyramid (LP) and the directional filter banks (DFB). The LP part is first applied so that the point discontinuities are identified, and subsequently a directional filter bank is used to link and form linear structures from the mentioned point discontinuities. The contourlet transform has a redundancy ratio of less than 4/3[4].

Recently, a number of algorithms have used CT for image compression. In [4] and [5], the authors studied and analyzed the Non-Linear Approximation (NLA) capability of CT for low bit-rate image compression. Torcan et al. [6] propose a graph cut algorithm which uses rate-distortion optimization for coding of CT coefficients. In [7] wavelet is used in conjunction with CT and a SPIHT-like algorithm. Furthermore to reduce the redundancy of the CT in [8] and [9]a wavelet based CT (WBCT) algorithm and its enhancement are respectively presented. On the other hand, in [10] a new family of non-redundant transforms is proposed which are based on hybrid use of wavelets and directional filter banks (HWD).This method generally has comparable or better NLA performance as compared to CT [10].Generally, in NLA algorithms after performing an orthogonal transform of the image, M larger coefficients are stored and the rest of the coefficients are discarded. The reconstructed image will then be an approximation of the original one and is formed using the M stored coefficients [4]. In NLA based compression algorithms that use DFB, such as CT, WBCT, and HWD, due to the preservation of higher coefficients, usually the details of the image, based on the required bit-rate are preserved. But the low frequency details, and the smooth regions with few details, are affected with the pseudo-Gibbs phenomena artifacts. This is caused by setting some transform coefficients to zero in the DFB stage [10].

In this paper we propose a compression algorithm, which unlike NLA algorithms, modifies the coefficients in a controlled manner rather than rigid replacement of some of the coefficients with zeros. To illustrate our algorithm we have applied it to contourlet transform. The range of the applied modifications changes adaptively to apply small changes to important coefficients and to vary more the coefficients of lesser importance. This allows preservation of more details and minimization of entropy. The paper's organization is as follows. Section 2 presents the proposed algorithm and in section 3 the implementation results are presented. We conclude the paper in section 4.

2. PROPOSED ALGORITHM

In the presentation of the proposed algorithm it is assumed that the contourlet transform is applied to the input image. This causes the decomposition of the image into an approximate subband and several directional subbands at multiple scales. These coefficients are converted to integers. Hence, when we refer to a coefficient it has integer value. Then we change each coefficient$c$ by adding/subtracting a positive integer to/from it to generate a modified coefficient $\tilde{c}$. Hence

$$|c - \tilde{c}| \leq d_{max} \quad (1)$$

Using the constraint of Equation 1 none of the CT coefficients will be altered more than d_{max}. By doing so, unlike what happens in the NLA based algorithms, based on the value of d_{max} the lower frequency details of the image are preserved too.

Our algorithm first sifts the coefficients of each scale, independently, to separate the significant ones. Then impact of each scale is calculated and proportional to that the maximum allowed modification for each scale is found. Coefficients are modified to minimize the entropy and the resulting coefficients are compressed in a file. These steps are now elaborated.

2.1. Per scale significance of coefficients

Different coefficients resulting from the contourlet transform have different roles and impact in the reconstruction of the image. The proposed algorithm tries to modify the coefficients of a scale inversely proportional to the impact of that scale. Hence, for scales which have higher number of large coefficients, the modifications would be small. On the other hand, for scales with small number of large coefficients, larger modifications are performed. With this goal in mind, Equation (1) is rewritten as

$$\forall\ 0 \leq i \leq L, 1 \leq j \leq N_i,\ d_i \leq d_{max}\ ,\ \left|C_i^j - \tilde{C}_i^j\right| \leq d_i \quad (2)$$

where L is the number of multi-scale decomposition levels in CT and N_i is the number of coefficients in the ith scale. $C_i^j$and $\tilde{C}_i^j$are respectively the jth coefficient of the ith scale before and after modification. The alteration of the coefficients of the ith scale can be at most equal to d_i. In Equation (2) the case $i = L$ referes to the finest scale and $i = 0$ corresponds to the approximation subband. We calculate d_i, adaptively based on the number and magnitude of the *significant* coefficients in a subband.

The significance of a coefficient should be defined. Since in contourlet transform the number of coefficients and their magnitudes are different for each scale, it is not appropriate to use the energy as a criterion for the significance of the coefficients of that scale. Hence, a threshold T_i is found by applying Otsu's algorithm [11] to the absolute value of the coefficients of each scale. Coefficients larger than T_i are considered significant and will be involved in determining the *impact* that the scale has in the reconstruction of the image. Note T_0 that is taken as 0.

2.2. Modification-range computation

To calculate the $impact$ of the ith scale, the average energy of the significant coefficients of the scale is normalized by the magnitude of the largest coefficient of the ith scale as shown in Equation (3). In this equation N_{T_i} is the number of significant coefficients in scale i.

$$Impact_i = \frac{\sum_{\left|C_i^j\right| \geq T_i} (C_i^j)^2}{N_{T_i} \times max\left(\left|C_i^j\right|\right)} \quad (3)$$

Since C_0^j consists of approximate subband coefficients and has higher energy as compared to other scales, we use $Impact_0$ as the benchmark and compare the impact of other scales with it. The maximum modification that is applied to the coefficients of each scale is proportional to the impact of that scale. The farther the impact of the scale is from$Impact_0$ the larger will be the alterations applied to it. The distance between $Impact_i$ and $Impact_0$ is:

$$distance_i = Impact_0 - Impact_i \quad (4)$$

The maximum alterations allowed for the ith scale is d_i which is equal to d_{max} for the scale which has maximum distance from the approximation subband. This is shown in Equation (5). Also the amount of allowed alterations for the coefficients of the approximation subband is zero since $distance_0 = 0$.

$$d_i = \left\lfloor \frac{distance_i}{max(distance_i)} \times d_{max} \right\rfloor \quad (5)$$

2.3. Modification of coefficients

The altered coefficients in each scale are to be compressed hence the alterations, with the boundaries of Equation 2, will be done with the aim of reduction of the entropy of the modified coefficients in that scale. Modification of coefficients in a scale is equivalent to changing the histogram of the coefficients. Suppose the histogram of the original integer coefficients of the ith scale is denoted by H_i and the histogram of the modified coefficients is called $\widetilde{H}_i$. We proved in [12] that for $\widetilde{H}_i$ to have minimum entropy, either a histogram bin of H_i should be left intact or it should be transferred completely to another bin. It is expected that the content of many of the bins of $\widetilde{H}_i$ are zero.

When a coefficient with value g is to be modified by a certain amount, all of the coefficients in that scale with value g will be modified by the same amount. Then there are $L_i = g\_max_i - g\_min_i + 1$ bins in H_i where g_max_i and g_min_i are the largest and smallest CT coefficients of the ith scale. Considering $d_i = 1$, there are about 3^{L_i} possible ways to form $\widetilde{H}_i$. This means, that we have to combine every two or three bins together to form a $\widetilde{H}_i$ were number of non-zero bins is lower than H_i. An algorithm is now explained which indicates the bins that should be changed and the amount of change. Our graph based bin-grouping algorithm uses Viterbi [13] scheme to decide what bins need transformation, and in what way, so the final entropy of the coefficients is minimized [12].

The following explanation is for $d_i = 1$ but the algorithm can be used for any value of d_i. In section 3 we present results for the application of the algorithm with different values of d_{max}. A part of a graph formed for the histogram of the ith scale, as an example, is shown in Fig. 1. Each bin of the histogram (h_k, $g\_min_i \leq k \leq g\_max_i$) is represented by a node in the graph. An arc in the graph represents a grouping of a number of adjacent nodes. The group includes the node where the arc originates from, and all of the nodes that the arc jumps over. A path on the graph starts from the node at the very left of the histogram and goes through a number of nodes and will end at the node at the very right of the histogram. Therefore, every path represents a histogram of modified coefficients. Referring to Fig. 1, as an example, a path is shown with dotted arcs. We see that bins h_0 and, h_1 , and h_2 are to be grouped in one bin. In another words, $\tilde{h}_1 = h_0 + h_1 + h_2$, hence $\tilde{h}_0 = 0$ and $\tilde{h}_2 = 0$. Also we see that bins h_3 and h_4 are combining to form one bin which means that either $\tilde{h}_3 = 0$ and $\tilde{h}_4 = h_3 + h_4$ or $\tilde{h}_3 = h_3 + h_4$ and $\tilde{h}_4 = 0$. The resulting entropy from either of these two choices is the same. Now, a path should be selected (from a large collection of possible paths) to minimize the overall entropy of $\tilde{H}_i$. All of the arcs in the graph are then labeled with the local entropy that will result from the grouping of the bins that the arc indicates. Using the Viterbi algorithm we can select a path on this graph with minimum entropy. This algorithm can be used for any value of d_i [12].

2.4. Compression of coefficients

Modifications of the coefficients were performed to minimize the entropy. Suppose that function $most(i)$ finds the coefficient which occurs most frequently in scale i. To further increase the compression capability of the algorithm and to exploit the inherent spatial redundancy of the modified histogram, we take out coefficient $most(i)$ of the pool of the coefficients and all of the remaining coefficients are compressed by adaptive arithmetic coding. To correctly preserve the location of the coefficients, a mask is formed such that everywhere there is a $most(i)$ we have a zero and at the locations of other coefficients there will be a 1. This mask is compressed by first applying RLE (Run Length Encoding) and then adaptive arithmetic coding. Finally, $most(i)$ and the compressed data are stored.

3. IMPLEMENTATION RESULTS

In this section results from the implementation of the proposed algorithm are presented. It should be mentioned that our goal is to present a method for preserving the transform coefficients in such a way that the reconstructed image has high quality and efficient compression of the coefficients is possible. To generate a complete compression

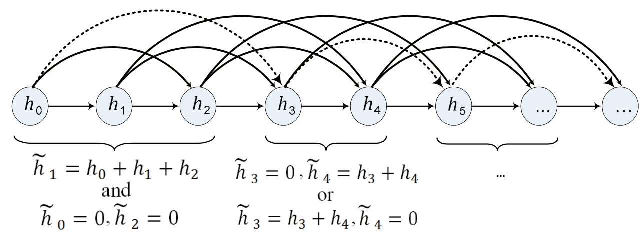
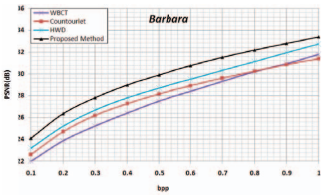


Fig. 1. An example of graph formation for $d_i = 1$.

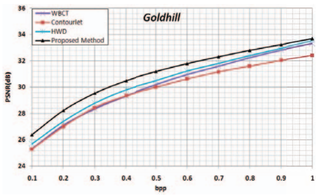
tool certainly needs other requirements. That is why we compare our method of coefficient preservation with those of NLA methods. To evaluate the performance of our algorithm we compared it with the results that we obtained from implementation of CT[4,5], WBCT [8], and HWD [10]. We used NLA method for generating the coefficients. Also we compressed the coefficients using a method similar to that mentioned in section 2.4, except it is applied to all of the coefficients and $most(i) = 0$. A five-level contourlet decomposition was used, where each of the three coarsest levels consists of a 9/7 separable wavelet transform, in three directions, and the finest two levels are respectively represented by 16 and 32 bands directional filters. In the DFB stage we used non-separable fan filters [14]. For both of WBCT and HWD, five wavelet levels have been used. Also eight directions at the finest level are used for WBCT and three directional levels have been selected for HWD.

We compared our proposed algorithm with the other mentioned algorithms using standard images. We compressed images for different values of d_{max} and measured the produced PSNRs. For example in Fig.2 the rate-distortion measurements are plotted for the *Barbara*, *Baboon*, *Goldhill,* and *Zoneplate* images. As can be seen for all of the mentioned images, at low bit rates, the proposed algorithm produced better PSNR, for similar bit rates, as compared with the CT based NLA method. We also compared our method with WBCT and HWD. The WBCT method is an image transform with a construction similar to the contourlet transform, but unlike CT, has no redundancy. Furthermore, HWD which has no redundancy has shown better NLA behavior as compared to CT [10]. Our method caused CT to perform better than both WBCT and HWD. Overall, the proposed method achieves 0.41 to 3.95 dB PSNR gain over CT,0.17 to 6.31 dB gain over WBCT, and 0.15 to 3.57 dB gain over HWD for the mentioned images at low bit rates.

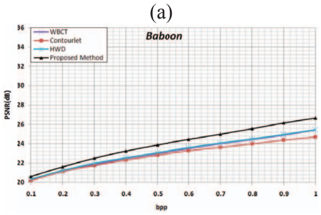
To compare the visual quality of images, Fig. 3 shows a portion of two original test images and the resulted images reconstructed by our algorithm and by the CT-based NLA algorithm. The reconstruction of the images was performed for the bit-rate of 0.25. It is shown in Fig. 3 that for *Barbara* the outcome of our algorithm produces an image that the periodic texture on her pants is very well preserved. In general, the textures in most parts of the image are very much undisturbed. Also the details of the shape of her shoulder and arm has low blurring while in the other algorithm the arm is blended with the background. In case



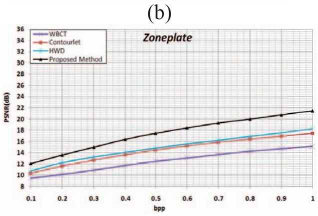
(a)



(b)



(c)



(d)

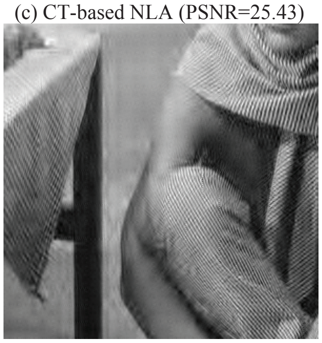
Fig. 2. Rate-distortion performance of the CT, WBCT, HWD and the proposed algorithm on tested images (a) *Barbara*. (b) *Goldhill*. (c) *Baboon*. (d) *Zoneplate*.

(a) Original image (*Barbara*)

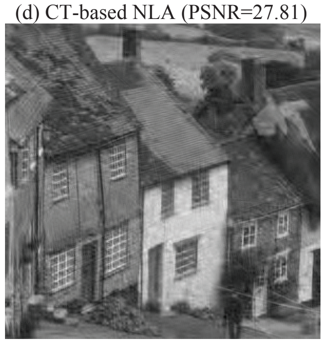
(b) Original image (*Goldhill*)

(c) CT-based NLA (PSNR=25.43)

(d) CT-based NLA (PSNR=27.81)



(e) Ours (PSNR=27.01)



(f) Ours (PSNR=28.95).

Fig. 3. The *Barbara* and *Goldhill* images coded at the bit- rate of 0.25 bpp.

of *Goldhill*, the CT-based NLA algorithm did not preserve the details of the houses nearly as much as our algorithm did. These details are more apparent for the tiles on the roofs, the details of the windows of the houses, and the background scenery. Our algorithm produced higher PSNR values as well as better visual qualities than the CT- based NLA algorithm not only for the highly textured *Barbara*, but also for the much smoother *Goldhill*. We also obtained similar results for the other two images.

4. CONCLUSION

In this paper we presented a general framework for modification of transform coefficients and we showed its application in the contourlet transform. Our algorithm has the advantage of adaptively altering the coefficients in such a manner that the overall entropy of the compressed coefficients is minimized while the details of the image are preserved. We showed that our modifications caused CT to perform better than more advanced algorithms in terms of both produced PSNRs and visual qualities.

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